Responsibility Centers, Decision Rights, and Synergies*

Tim Baldenius†  Beatrice Michaeli‡

August 1, 2019

*We thank two anonymous referees and Brian Cadman (the Editor), as well as Wouter Dessein, Henry Friedman, Mirko Heinle, Thomas Hemmer, John Hughes, Raffi Indjejikian, Scott Keating, Volker Laux, Thomas Pfeiffer, Stefan Reichelstein, Tatiana Sandino, and seminar participants at MIT, Harvard Business School, University of Utah, 2018 UCLA Spring Conference, UIUC Emerging Scholars Conference, INSEAD Accounting Symposium, University of Vienna, and 13th Workshop on Accounting and Economics (Paris) for helpful comments and suggestions.

†Columbia Business School, tim.baldenius@columbia.edu

‡Anderson School of Management, UCLA, beatrice.michaeli@anderson.ucla.edu
Abstract

Responsibility Centers, Decision Rights, and Synergies

We consider the optimal allocation of decision rights over noncontractible specific investments. Risk-averse business unit managers each engage in general (stand-alone) operations and invest in joint projects that benefit their own and other divisions. Which of the managers should have the authority to choose these investments? With scalable investments, we show that decision rights should be bundled in the hands of the manager facing the more volatile environment. With discrete (lumpy) investments, on the other hand, decision rights should be split between the managers, provided they face comparable levels of uncertainty in their general operations. Splitting decision rights better leverages the inherent investment complementarity, counter to conventional wisdom. Our model generates empirical predictions for the equilibrium association of organizational structure and managers’ incentive contracts: bundling of decision rights results in pay-performance sensitivity (PPS) divergence across divisions; splitting them results in PPS convergence.

Keywords: responsibility centers, task allocation, pay-performance sensitivity, investments, risk

JEL codes: M41, D23, D86
1 Introduction

How should decision rights over synergistic investments, that affect more than one division within a firm, be assigned to the units? Should they be distributed evenly or bundled in the hands of a single division manager, making that division an investment center and the other(s) mere profit centers? The management literature invokes information or coordination needs as key determinants of this design choice (e.g., Sosa and Mihm 2011): if dispersed information is important, and such information cannot easily be shared, then decision rights should be decentralized; otherwise, bundling improves coordination. By contrast, we argue in this paper that even in the absence of informational frictions among the divisions involved in a joint project, splitting decision rights between them can be advantageous.

To address this question, we study a setting with two business units that are engaged in their general (stand-alone) operations and collaborate on a project. The efficiency of the project can be enhanced by upfront specific investments. For example, in a supply chain, an upstream investment (e.g., in product design) may reduce the variable production cost; a downstream investment (e.g., in marketing) may increase the customers’ willingness to pay. We adopt an incomplete contracting approach by assuming both ex-ante investments and ex-post proceeds are noncontractible at the project level. But we allow for the principal to assign decision rights over investments to the business units, and we ask whether each division manager should be in charge of one dimension of the investment choice (the split regime), or whether one manager should choose both (bundling)—and if the latter, which manager?1

1 Examples are the assignment of process improvement projects to certain business units when multiple units later share in the benefits of these projects, or the curriculum development investments undertaken by business and engineering schools within a university to launch interdisciplinary degree programs. Joshi and Gimenez (2014) provide examples for the assignment of decision rights across marketing, IT, and analytics departments at Nordstrom and Target: “At Target, category marketers...specify...what the decision rights are for each decision, and what inputs each participant should contribute” (p.7); “[R]assigning decision rights and clar-
We show that scalable (continuous) investments should be bundled in the hands of the manager facing greater general uncertainty. High pay-performance sensitivity (PPS) elicits general-purpose effort but reduces a manager’s incentive to make specific investments, as such investments add not only to the expected surplus of the project but also to its variance (Baldenius and Michaeli, 2017). High-powered incentives make a manager more sensitive to the incremental project risk resulting from the investment. The manager facing a more volatile environment will have more muted PPS—based on standard contracting arguments—and thus greater induced risk tolerance. As a result, he is more willing to invest, which mitigates the hold-up problem associated with surplus sharing (Williamson, 1975). Moreover, bundling benefits from a “two-for-one” effect, as lowering the PPS of the investment center manager under bundling stimulates both investments directly.

In many cases, project investments are lumpy—e.g., replacement of existing equipment, M&A, new-product development, or entry to a new market. Our results then change qualitatively: in particular, decision rights over lumpy project investments should be split between the managers if their general uncertainty levels are not too different. The reason is: (i) specific investments naturally tend to be strategic complements—the resulting efficiency improvement increases the optimal project scale, which in turn raises the marginal return to other investments; (ii) lumpiness amplifies this strategic complementarity; and (iii) splitting decision rights better leverages this complementarity. Absent risk considerations, complementary lumpy investments are elicited more cheaply as a Nash equilibrium of a non-cooperative game (split regime) than from a single decision maker (bundling). The split regime requires investing be each player’s best response

ifying roles for the marketing, analytics, and IT functions has ensured that Nordstrom can rapidly enhance vital tools and technology capabilities” (p.9). Related anecdotes are reported also for joint projects in intra-firm settings, e.g., Asanuma (1989) reports how “specific investments...have to be incurred to implement...customization” of parts for buyers by suppliers, while Nishiguchi (1994) describes how suppliers “send engineers to work...in design and production...play innovative roles in...gathering information about long-term product strategies.”
to the other player investing; bundling in contrast requires the investment center manager to prefer investing two units (and paying for both fixed costs) to not investing at all—strategic complementarity makes this the more demanding condition. This insight runs counter to the conventional wisdom that decisions with externalities should be bundled, as expressed by the coordination theme in our opening paragraph.

The principal’s optimal regime choice with lumpy investments is driven by a tradeoff between strategic complementarity (in favor of the split regime) and the induced risk tolerance effect (in favor of bundling). If the managers face operating environments that are subject to similar levels of uncertainty, the risk tolerance effect will be small, and the split regime is preferred. But if the managers face very different levels of general uncertainty, bundling decision rights in the hands of the manager facing the more volatile operations is again preferred.2

Our analysis sheds light also on the equilibrium association of organizational structure and incentive strength across business unit managers: Bundling of decision rights results in incentive divergence across divisions; the split regime results in incentive convergence. Under bundling, the high-uncertainty (low-PPS) manager is assigned investment authority—it is his PPS that needs to be lowered further to stimulate investments. Under the split regime, the low-uncertainty (high-PPS) manager is the bottleneck—his PPS has to be lowered first to provide investment incentives. While we assume decision rights can be moved across divisions at no direct cost, this may not always be descriptive. In settings where a firm is stuck with the split regime, for technological reasons, our model thus predicts PPS compression. In contrast, if tasks can be freely allocated, and they are scalable, our model predicts greater disparity in variable

2Throughout the paper, we will contrast our results with those obtained in a standard “pure hold-up” setting, i.e., abstracting from the investment-risk link, e.g., Edlin and Reichelstein (1994), Baldenius et al. (1999), Pfeiffer et al. (2011). In the pure hold-up setting the PPS has no effect on investments. While this would render the principal’s optimal regime choice vacuous for scalable investments, for lumpy investments the split regime would always dominate, because the strategic complementarity effect would persist.
Table 1: Optimal allocation of decision rights

<table>
<thead>
<tr>
<th>Nature of project inputs</th>
<th>Monetary investments</th>
<th>Personally costly efforts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalable</td>
<td>Bundling decision rights in the hands of the manager who faces high general uncertainty</td>
<td>Bundling decision rights in the hands of the manager who faces low general uncertainty</td>
</tr>
<tr>
<td>Lumpy</td>
<td>Splitting of decision rights across managers</td>
<td>Bundling decision rights in the hands of the manager who faces high general uncertainty</td>
</tr>
</tbody>
</table>

compensation across business units, as tasks will then be bundled.

As a robustness check, we consider project-specific inputs that are personally costly to the manager who chooses them ("project efforts"). This scenario is descriptive of startups or knowledge-based firms where managers’ inputs often take the form of skill acquisition. Changing the nature of specific inputs flips the relation between incentives and equilibrium input levels: standard moral hazard arguments then apply, making each manager’s project effort an increasing function of his PPS.[3] Nonetheless, our results remain qualitatively intact except that, conditional on bundling, decision rights over project efforts should now be bundled in the hands of the manager facing the less volatile general operations (and thus higher-powered incentives). Table 1 summarizes these findings.

It is useful to clarify the driving forces behind our findings. The result that scalable monetary investments should be bundled in the hands of the high-uncertainty manager builds on the investment-risk link in Baldenius and Michaeli (2017). The result that scalable personally costly project efforts should be bundled in the hands of the manager facing the more stable operations is related to

---

[3] The investment-risk link—or more generally, input-risk link—is present also for personally costly project efforts. But this second-moment effect is outweighed by the first-moment effect, as in standard moral hazard models, that the PPS now scales up the manager’s internalized share of the project proceeds, without affecting how he internalizes the input (effort) costs.
the multitasking literature, in that high-powered PPS elicits different dimensions of efforts in tandem—some yielding only local benefits, others benefitting other divisions synergistically. The result that decision rights over lumpy (monetary or personally costly) inputs should sometimes be allocated evenly among the managers contrasts with earlier calls for bundling of authority in the presence of complementarities, e.g., Brynjolfsson and Milgrom (2012, p.13). The main reason for this departure is that, in our model, (i) the managers always split the gross project returns ex post and (ii) the manager that chooses an input also has to “pay” for it. These assumptions seem natural in incomplete contracting settings. Surplus sharing sets our model apart from agency papers that allow for more complete contracts to divide the output, e.g., Zhang (2003), Hughes et al. (2005), while the linkage of decisions rights and cost charges is distinct from the literature on authority, e.g., Dessein et al. (2010).

Our paper contributes to the literature on multitasking and task allocation. While Baiman et al. (1995) study the optimal degree of delegation between headquarters and a business unit manager, we take delegation as given and focus instead on the allocation of decision rights among managers. Bushman et al. (1995) and Edmans et al. (2013) assume that an agent’s action affects the productivity of other agents, while in Liang and Nan (2014) and Friedman (2014) it affects the variance of performance measures. In our model both effects arise endogenously—from first principles—as a result of specific investments increasing the efficient scale of cross-divisional “projects.” While the task allocation literature typically assumes that the assignment of decision rights affects the way the benefits from agents’ actions are measured (e.g., Holmstrom and Milgrom, 1987),

4For a survey of the authority literature see Bolton and Dewatripont (2012). Unlike that literature, in our model the investment center manager has no authority over the actions of the other manager: “authority is a supervisor’s power to initiate projects and direct subordinates to take certain actions” (Bolton and Dewatripont, 2012, p. 343).

5In multitasking settings, Autrey et al. (2010) study the determinants of agency costs due to aggregation and Heinle et al. (2012) discuss behavioral incentives. In Reichmann and Rohlffing-Bastian (2013) and Hofmann and Indijiekian (2017), the allocation of tasks or contracting power is delegated to lower hierarchical levels.
1990), this is not the case in our model, where the managers always share equally the project returns, irrespective of the allocation of decision rights. This sharpens our focus on the (novel) game form effect: creating a non-cooperative game by splitting tasks among managers better leverages input complementarities.

The paper proceeds as follows. Section 2 describes the model and the benchmark case of contractible investments. Section 3 and Section 4 consider the optimal allocation of decision rights with scalable and lumpy monetary investments, respectively. Section 5 extends the results to personally costly project-specific efforts. Section 6 concludes.

2 Model

Consider two division managers, \( i = A, B \). Each manager exerts general (operating) effort, and together they implement a joint project. The return to general effort, \( a_i \in \mathbb{R}_+ \), is normalized to one; Manager \( i \)’s personal effort cost is \( v a_i^2 \), \( v > 0 \). The joint project creates a (gross) surplus \( M(q, \theta, k) \), which depends on the project scale, \( q \in \mathbb{R}_+ \), a random state of nature, \( \theta \in \mathbb{R}_+ \), and relationship-specific investments, \( k \equiv (k_A, k_B) \), with \( k_i \) chosen from the set \( \mathcal{K}_i \) at a fixed cost of \( F(k_i) = \frac{f}{2} k_i^2 \). Let \( \mathcal{K} = \mathcal{K}_A \times \mathcal{K}_B \). We will consider both scalable investments (\( \mathcal{K}_i = \mathbb{R}_+ \)), in which case assuming \( f > 6 \) ensures global concavity, and lumpy investments (\( \mathcal{K}_i = \{0, 1\} \), without loss of generality). The setting builds on the single-investment model in Baldenius and Michaeli (2017) by adding an organizational design angle: we ask whether decision rights over the investment vector \( k \) should be bundled in the hands of one manager or split between the managers.

We assume the project surplus is \( M(q, \theta, k) = (\theta + k_A + k_B)q - \frac{q^2}{2} \), which in turn can be derived from a standard linear-quadratic supply chain setting.  

---

6Task allocation only affects the measurement of the costs of monetary project investments in our model.

7We follow Aghion and Tirole (1997), Hart and Holmstrom (2002), Bester and Krahmer (2008) in assuming that decisions themselves may not be contractible, but decision rights are.

8Suppose an upstream unit makes \( q \) units of an intermediate good at variable cost \( C(q, \theta_A, k_A) = (c - \theta_A - k_A)q \). The downstream unit sells a final product at revenues
Before observing $\theta$, the managers choose the investments. None of our results hinges qualitatively on the symmetry restriction that investments are equally productive. Investments and the state $\theta$ are jointly observable to the managers but cannot be communicated to the principal.

After observing $\theta$, the managers implement the project under symmetric information by agreeing on the efficient project scale, $q^*(k, \theta) = \theta + \sum_i k_i$, which maximizes $M(q, \theta, k)$. Denote the resulting value function by $M(\theta, k) \equiv M(q^*(\cdot), \theta, k) = \frac{1}{2}(\theta + \sum_i k_i)^2$. With equal probability, the random state variable $\theta$ takes values $(\mu - \sqrt{\eta})$ or $(\mu + \sqrt{\eta})$, with $\sqrt{\eta} < \mu$, so that $E[\theta] \equiv \mu$ and $Var(\theta) \equiv \eta$. The variance of the project surplus then simplifies to $\varphi(\kappa) \equiv Var(M(\theta, \kappa)) = (q^*(\mu, \kappa))^2 \eta$, so that $\varphi(\kappa)$ is increasing in each $k_i$, with increasing differences in $k$.

Specific investments make the joint project more efficient at the margin and thereby increase the project scale pointwise, for any $\theta$-realization. Ex ante, however, each expected unit of the project is subject to the random shock $\theta$. Hence, specific investment scales up the surplus variance. Baldenius and Michaeli (2017) refer to this as the investment-risk link.

Throughout the paper, we assume the project surplus $M(\cdot)$ is nonverifiable to the principal; instead, the managers negotiate over it among themselves. We assume they equally divide $M$, resulting in divisional income of

$$\pi_i = a_i + \varepsilon_i + \frac{M(\theta, k)}{2} - FC_i(k),$$

where $\varepsilon_i$ is an additively separable random shock to the general environment of Division $i$ with mean zero and variance $\sigma_i^2$, and $FC_i(k)$ is Division $i$’s fixed cost, to be specified below.$^9$ We label $\sigma_i^2$ Division $i$’s general uncertainty and $\eta$ the

$^9$Baldenius and Michaeli (2017) show that investment distortions are non-monotonic in the players’ bargaining power: a manager who has full bargaining power vis-a-vis his counterpart always underinvests, but he may overinvest if the surplus is shared equally. Assuming equal-split sharing allows us to sidestep these complications and focus on decision rights. In much
project uncertainty. Without loss of generality, we rank the general uncertainty levels such that $\sigma_A^2 > \sigma_B^2$, i.e., Division A faces a greater general uncertainty. All random variables, $\theta$ and $\varepsilon_i$, are independent.

Divisional income $\pi_i$ is contractible in aggregate but cannot be disentangled by the principal into its components. We confine attention to linear contracts and divisional performance evaluation, $s_i(\pi_i) = \alpha_i + \beta_i \pi_i$, where $\alpha_i$ is the fixed salary (or intercept), and $\beta \equiv (\beta_A, \beta_B) \in [0,1]^2$ are the managers’ pay-performance sensitivities (PPS). The managers have mean-variance preferences, $EU_i = E[s_i(\cdot)] - \frac{\nu}{2}a_i^2 - \frac{\rho}{2} \text{Var}(s_i(\cdot))$, where $\rho > 0$ is their (common) coefficient of risk aversion. Manager $i$’s expected utility then reads

$$EU_i(\cdot) = \alpha_i + \beta_i \left( a_i + \frac{E[M(\theta, k)]}{2} - FC_i(k) \right) - \frac{\nu}{2}a_i^2 - \frac{\rho}{2}\beta_i^2 \left( \sigma_i^2 + \varphi(k) \right). \quad (2)$$

To ensure the managers’ participation, we impose the individual rationality condition that $EU_i \geq 0$, for any $i = A, B$. Moreover, the principal observes the general effort incentive constraints,

$$a_i(\beta_i) \in \arg \max_{a_i} EU_i(\cdot) \quad (3)$$

Given this setting, each manager’s general effort choice is a function only of his own PPS; specifically, it does not depend on any investment choices. With fixed

of the prior hold-up literature, the negotiating managers were assumed to be risk neutral and residual claimants for their respective divisional profits. Here, they are risk-averse and pocket only their pay-performance sensitivity share of divisional profit. Therefore, equal split would no longer be the equilibrium outcome of Nash bargaining—but it is the equilibrium of an alternating offers bargaining game (Rubinstein, 1982) with a risk of breakdown, if the managers are equally patient and make offers in quick succession.

For tractability, we restrict attention to linear contracts based on divisional performance. Baldenius and Michaeli (2017, 2019) consider contracts that are nonlinear or that feature firmwide performance evaluation, respectively. The main driver of managerial compensation in practice is divisional profit, e.g., Merchant (1989), Bushman, et al. (1995), Keating (1997), Abernethy, et al. (2004), Bouwens and van Lent (2007), Bouwens, et al. (2018). Assigning the same incentive weight to own- and other-division’s income would be suboptimal in terms of risk sharing (Holmstrom and Tirole, 1991; Anctil and Dutta, 1999) and fails to induce contractible benchmark investments (Baldenius and Michaeli, 2019).

The assumption $\beta_i \in [0,1]$ ensures the principal has no incentives to destroy output and the managers have incentives to provide general efforts.

See Li (2018) for a structural estimation of management compensation assuming identical risk aversion across managers.
Contracts signed \(a_i\) and \(k_i\) \(\theta\) observed by managers Managers implement joint project and split \(M\)

\begin{figure}[h]
\centering
\begin{tabular}{c|c|c|c|c}
0 & 1 & 2 & 3 & 4 \\
\hline
Decision rights & Contracts & \(a_i\) and \(k_i\) & \(\theta\) observed & Managers implement joint project and split \(M\) \\
assigned & signed & chosen & by managers & \\
\end{tabular}
\caption{Timeline}
\end{figure}

salaries \(\alpha_i\) set to extract all surplus, the firmwide expected surplus is

\[
W(k, \beta) \equiv E[M(\theta, k)] + \sum_{i=A,B} \left[ a_i(\beta_i) - \frac{v}{2}(a_i(\beta_i))^2 - FC_i(k) - \frac{\rho}{2}\beta_i^2 \left( \sigma_i^2 + \frac{\varphi(k)}{4} \right) \right].
\] (4)

The timeline is given in Figure 1. At the outset, the principal assigns the investment decision rights and contracts with the managers. The managers choose their investment and general effort levels. The state of nature is realized, the project is implemented and the payoffs realized.

While we assume throughout that the surplus is nonverifiable ex post at the project level, we consider as a benchmark the case of contractible investments ex ante: The principal instructs the managers how much to invest at Date 2, and the managers jointly implement the project and split \(M(\cdot)\) at Date 4. Investments and the PPS are given by (superscript “*” indicates the contractible benchmark):

\[
(k^*, \beta^*) \in \arg \max_{\beta \in [0,1]^2, k \in K} W(k, \beta).
\] (5)

To assume well-behaved investment problems with interior solutions (for scalable investments), we assume throughout that the project uncertainty is bounded from above: for \(\eta_{\text{risk}} \equiv 4\sigma_B^2(1 - 2/f)^2/\mu^2\) and \(\eta_{\text{pos}} \equiv 1/\rho\),

\[
\eta \leq \min \{\eta_{\text{risk}}, \eta_{\text{pos}}\}.
\] (6)

Assuming \(\eta \leq \eta_{\text{risk}}\) ensures the project risk for each manager is less than his operating risk; for high project risk, one would expect the divisions to be merged.
The restriction $\eta \leq \eta_{pos}$ ensures that (i) the pressing concern is underinvestment and (ii) the principal chooses positive benchmark investment levels, $k_i^* > 0$.\footnote{For cases in which overinvestment may arise, see Baldenius and Michaeli (2017).}

In the main setting, investments are noncontractible. We ask the classic organization design question, how to assign decision rights over the Date-2 investments to the managers. Under the split regime, decision rights are allocated equally: each division is organized as an investment center, and Manager $i$ chooses $k_i$ (Figure 2a).\footnote{Given our assumption that the investments are equally productive, it is without loss of generality under the split regime that Manager $i$ chooses $k_i$, $i = A, B$, rather than $k_j$, $j \neq i$.} Under bundling, in contrast, the principal assigns all investment decision rights to some Manager $\ell \in \{A, B\}$, who chooses both $k_A$ and $k_B$; the other manager has no investment authority whatsoever, his unit is run as a profit center (Figure 2b).

Irrespective of the regime, both business units remain essential at the Date-4 project implementation stage. That is, the managers continue to share the surplus $M(\cdot)$ equally ex post, even if ex-ante investment authority is bundled. (At Date 4, the fixed investment costs are sunk and will have no effect on surplus splitting.) Note that these two regimes cover the spectrum of allocations of decision rights; a horse race between them thus will deliver the optimal regime, as there is no benefit to randomization here.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{organizational_design.png}
\caption{Organizational design}
\end{figure}
We build on earlier studies of settings in which decisions themselves may not be contractible, but decision rights are, e.g., Aghion and Tirole (1997), Hart and Holmstrom (2002), Bester and Krahmer (2008). Note that whenever investments are noncontractible, fixed cost charges are tied to decision rights: if Manager \( i \) has the authority to choose \( k_j \), then the fixed cost \( F(k_j) \) will reduce his divisional income measure, \( \pi_i \), that is:

\[
FC_i(k) = \begin{cases} 
F(k_i), & \text{under the split regime}, \\
(F(k_A) + F(k_B)) \times 1_{i=\ell}, & \text{under bundling with } \ell = i,
\end{cases}
\]

where \( 1_{i=\ell} \in \{0, 1\} \) is the indicator function. Because the managers split the project surplus equally, irrespective of the regime choice, we can restate Manager \( i \)'s gross expected payoff from the project (ignoring the fixed cost and omitting project-irrelevant terms from (2)) as\(^{15}\)

\[
\Gamma(k | \beta_i) \equiv \beta_i \left( \frac{E[M(\theta, k)]}{2} - \frac{\rho}{8} \beta_i \phi(k) \right).
\]

Under the split regime, at Date 2, the managers choose their respective investments simultaneously as a pure-strategy Nash equilibrium: for given \( \beta \),

\[
\max_{k_i \in K_i} \Gamma(k | \beta_i) - \beta_i F(k_i), \quad i, j = A, B, \quad i \neq j.
\]

Denote the (subgame) equilibrium investments for given \( \beta \) by \( k^S(\beta) \), where the superscript “S” denotes the split regime. At Date 1, the principal anticipates the Date-2 investment subgame outcome and solves

\[
\text{Program } P^S : \max_{\beta \in [0,1]^2} W(k^S(\beta), \beta).
\]

We assume an interior solution and denote it by \( \beta^S = (\beta^S_A, \beta^S_B) \). The equilibrium investments under the split regime are \( k^S \equiv (k^S_A, k^S_B) \equiv k^S(\beta^S) \).

\(^{15}\)There is no need to index the \( \Gamma(\cdot) \)-function because the managers are identical except for their general uncertainty. Also, in our model investments are of equal productivity, and thus \( \Gamma(x, y | \beta_i) \equiv \Gamma(y, x | \beta_i) \), for any \( x, y, \beta_i \).
Under bundling, Manager $\ell$ is the designated investment center manager. At Date 2, he chooses both investments, where the superscript “$\ell$” denotes bundling (with Manager $\ell$ choosing both investments):

$$k^\ell(\beta) \in \arg \max_{k \in K} \Gamma(k_A, k_B \mid \beta_{\ell}) - \beta_{\ell} \sum_{i=A,B} F(k_i).$$

(9)

A key difference to the split regime is that the profit center manager’s PPS, $\beta_j \neq \ell$, no longer affects any investments. The principal’s Date-1 contracting problem for given allocation of decision rights, $\ell$, reads

**Program $P^\ell$:** \[ \max_{\beta \in [0,1]^2} W(\beta, k^\ell(\beta)). \]

Denote the solution to this program by $\beta^\ell$ and the resulting investments by $k^\ell \equiv (k_{A}^\ell, k_{B}^\ell) \equiv k^\ell(\beta_{\ell})$. To level the playing field between the regimes, we abstract from (dis-)economies of scope in terms of the total investment cost.

At Date 0, the principal asks which of the two bundling programs $\hat{\mathcal{P}}^{\ell=i}$, $i = A, B$, delivers a higher value—i.e., which division to make the investment center—and then compares the achievable surplus with that under the split regime. As we now show, the nature of the investments (scalable or lumpy) critically affects the optimal regime choice.

### 3 Scalable Investments

We begin by assuming investments are perfectly scalable, i.e., $\mathcal{K} = [0, \bar{k}]^2$. Starting with the benchmark case of contractible investments, it is easy to see that the conditionally optimal PPS for given investments $k$ is

$$\beta_i^{o}(k) = \left(1 + \rho v \left[\sigma_i^2 + \frac{\varphi(k)}{4}\right]\right)^{-1}.$$  

(10)

Let $\beta^{o}(k) = (\beta_A^{o}(k), \beta_B^{o}(k))$. Now, the optimal investment, $k^* \in [0, \bar{k}]^2$, maximizes the value function $W^*(k) \equiv W(\beta^{o}(k), k)$; hence, $\beta_i^* = \beta_i^{o}(k^*)$. Because the investments have identical effects on fixed costs and risk premia, by (4), the
benchmark investments are identical: \(k^*_A = k^*_B\). The investment-risk link implies that \(\beta^*_i(k)\) is decreasing in \(k_j\), for any \(i, j\). Accounting for the project risk reduces the PPS below \(\beta^{MH}_i = (1 + \rho \sigma^2_i)^{-1}\), which is the bonus coefficient in a pure moral hazard model without joint projects. Thus, investments and PPS are substitutes for the principal in the benchmark case: greater investments add to the marginal risk premium, hence calling for muted incentives, and vice versa. By \(\sigma^2_A > \sigma^2_B\), we have \(\beta^*_A \leq \beta^*_B\). The maintained assumption \(\ref{eq:6}\) implies that \(\lim_{\eta \rightarrow \eta_{risk}} \beta^*_i = \beta^*_i \equiv (1 + 2\rho \sigma^2_i)^{-1}\) and, therefore, \(\beta^*_i \in [\beta^*_i \equiv (1 + 2\rho \sigma^2_i)^{-1}, \beta^{MH}_i]\).

In earlier incomplete contracting models that have ignored project risk, bilateral investments tend to be mutually reenforcing: efficiency-enhancing investments by one manager increase the expected project scale, which in turn raises the marginal investment return to the other manager, and vice versa. That is, the firmwide expected contribution margin, \(E[M(\cdot)]\), displays increasing differences (complementarity) in the investments. Although the same is true for the managers’ project-related risk premium, given the bounds on the inherent project uncertainty, as per \(\ref{eq:6}\), it is easy to show that the first-moment effect dominates, making contractible investments complements at the margin:

**Lemma 1** For given PPS, \(\beta\), investments are complements not only for the principal in the benchmark case, but also for each individual manager under decentralization—that is, for any \(\beta\), both \(W(k, \beta)\) and \(\Gamma(k \mid \beta_i)\) have strictly increasing differences in \(k\).

Given the assumed fixed cost separability, the strategic complementarity property for the managers’ gross investment return functions, \(\Gamma\), carries over to their net investment returns as stated in \(\ref{eq:8}\) and \(\ref{eq:9}\). We now turn to the managers’ investment incentives under the split regime.

**Lemma 2** For scalable investments \((k \in [0, \bar{k}]^2)\) and \(i, j = A, B\), under the split regime:
(a) For given $\beta$, there exists a unique equilibrium investment profile with $k^S_i(\beta) = \frac{\mu(2-\rho \eta \beta)}{\rho \eta (\beta_i+\beta_j)+4(f-1)}$, $j \neq i$. Each investment level is decreasing in the PPS of either manager: $\frac{dk^S_i(\beta)}{d\beta_j} < 0$.

(b) With endogenous contracts, both managers underinvest ($k^S_i < k^*_i$), but $k^S_A > k^S_B$.

A manager’s performance measure scales all divisional cash flows—including the fixed cost—equally by his PPS. Hence, the sole first-order effect of an increase in PPS is that the manager will be reluctant to invest as he will be more exposed to the investment-related risk. Strategic complementarity reinforces this effect and implies that each manager will invest less, also, the greater is his counterpart’s PPS.\textsuperscript{16} Comparing across divisions, in equilibrium, Manager A will invest more than Manager B under the split regime because $\sigma^2_A > \sigma^2_B$. Greater general uncertainty is associated with lower PPS for Manager A, making him more tolerant to the incremental investment-related project risk. Yet even Manager A will underinvest relative to the benchmark level because of the hold-up problem associated with ex-post surplus sharing.

We now turn to bundling, where Manager $\ell$, the designated investment center manager, chooses both $k_A$ and $k_B$. As argued in connection with the optimization problem in (9), Manager $\ell$’s choice of investments is affected only by his own PPS. In contrast to the split regime, therefore, the resulting investment profile under bundling is always symmetric: $k^\ell_A = k^\ell_B$, for any $\ell$. The arguments in Lemma 2 for underinvestment and the investment-suppressing effect of the PPS apply with only minor modifications to bundling (proof omitted):

**Lemma 3** For scalable investments ($k \in [0, \bar{k}]^2$), under bundling:

(a) For given $\beta$, Manager $\ell$ chooses $k^\ell_A(\beta) = k^\ell_B(\beta) = \frac{\mu(2-\rho \eta \beta)}{2\rho \eta \beta_i+4(f-1)}$. Each investment $k^\ell_i(\beta)$ is decreasing in the PPS of the investing manager $\beta_i$.

\textsuperscript{16}As one would expect, the closed-form term for $k^S_i(\beta)$ indicates that Manager $i$ responds more sensitively to changes in his own PPS, $\beta_i$ (the direct interaction between $k_i$ and $\beta_i$), than to changes in $\beta_j$ (the indirect effect through investment complementarity).
(b) With endogenous contracts, for any $\ell, i$, Manager $\ell$ underinvests ($k^\ell_i < k^*_i$).

We now ask which regime maximizes the principal’s expected payoff. A remaining design issue for the principal under bundling is, in whose hands to concentrate the decision rights—that is, which of the managers to designate the investment center manager (whether to set $\ell = A$ or $\ell = B$):

**Proposition 1** For scalable investments ($k \in [0, \bar{k}]$), bundling with $\ell = A$ (high general uncertainty) dominates both bundling with $\ell = B$ and the split regime.

The hold-up problem causes underinvestment for either delegation regime—but which regime is most effective in alleviating this distortion? With scalable investments, the answer unambiguously favors *bundling investment authority in the hands of the manager who faces greater general uncertainty*. Having lower PPS, all else equal, this manager is more tolerant toward the investment-induced project risk, and thus more willing to invest. Put differently, the key to stimulating delegated investment is muted PPS, and the attendant opportunity cost—foregone effort—is minimized by designating Manager A the investment center manager. Moreover, bundling offers a “two-for-one” benefit: lowering the PPS of the investment center manager stimulates both dimensions of investment directly, whereas lowering a manager’s PPS under the split regime directly stimulates only one dimension, at the same cost of foregone effort.

The two-for-one investment benefit of bundling would arise even if the general uncertainty levels $\sigma^2_i$ were to converge, making the induced risk tolerance effect disappear. However, if the investment-risk link were absent, say because $\rho \to 0$ so that the PPS approaches unity, we would recoup the standard “pure hold-up” model\[^{17}\] The PPS then would have no effect on investments; hence for scalable investments, there would be no difference between the regimes.

We now turn to the equilibrium contracts. Even with bundling ($\ell = A$) as the principal’s optimal regime choice, some amount of underinvestment will

\[^{17}\text{E.g., Edlin and Reichenstein (1994), Baldenius et al. (1999), Pfeiffer et al. (2011).}\]
persist (Lemma 3b). This reduces the project-related risk premium, which in turn calls for higher-powered PPS for either manager, all else equal. On the other hand, muting \( \beta_A \) would alleviate the underinvestment problem. Baldenius and Michaeli (2017) have studied this tradeoff for the special case of one-dimensional investment—their result generalizes in a straightforward manner to our setting in which Manager A chooses two dimensions of investment (proof omitted):

**Corollary 1** For scalable investments \((k \in [0, \bar{k}]^2)\), in equilibrium (i.e., bundling with \(\ell = A\)), Manager A faces lower-powered PPS than in the contractible benchmark if \(\sigma_A^2 > \sigma_{A,oo}^2\), but higher-powered PPS if \(\sigma_A^2 < \sigma_{A,oo}^2\) and \(f > f^o\), for some finite and feasible thresholds \(\sigma_{A,oo}^2 \geq \sigma_{A,oo}^2\) and \(f^o > 0\). Manager B always faces higher-powered PPS than in the benchmark.

Because the investment center manager (Manager A) operates in a volatile environment, his PPS will be low, all else equal. This dampens the risk premium effect, and it raises the sensitivity with which his choice of investments will react to changes in \(\beta_A^{18}\) hence the call for muted PPS. If \(\sigma_A^2\) is small and the fixed costs sufficiently convex, the investment inducement effect is small. The investment center manager then should be given a higher-powered incentive contract.

In either case, it is easy to see that incomplete contracting on scalable investments results in further PPS divergence across the divisions, i.e., \(\beta_B^{t=A} - \beta_A^{t=A} > \beta_B^o - \beta_A^o\). Equilibrium underinvestment allows for higher-powered incentives for both managers. At the same time, to foster investments, Manager A—whose PPS was the lower one to begin with, by standard volatility arguments—sees his incentives further muted relative to \(\beta_B^{19}\).

---

18 The sensitivity argument holds because investments are not only decreasing in the PPS, but this relationship is convex; see Baldenius and Michaeli (2017, Proposition 1) for details.

19 More formally, the effect of incomplete contracting on Manager B’s PPS is given by \(\beta_B^{t=A} - \beta_B^o = \beta_B^o(k^{t,A}) - \beta_B^o(k^*)\), because his PPS will be adjusted to the one that is optimal conditional on the equilibrium investments \(k^{t,A}\). The corresponding effect on Manager A’s PPS is bounded from above in that \(\beta_A^{t=A} - \beta_A^o < \beta_A^o(k^{t,A}) - \beta_A^o(k^*)\), because the investment inducement effect implies that \(\beta_A^{t=A} < \beta_A^o(k^{t,A})\), where \(k^{t,A} < k^*\). Lastly note that \(\beta_B^o(k_1) - \beta_A^o(k_1) > \beta_B^o(k_2) - \beta_A^o(k_2)\), for any two investment vectors \(k_1 > k_2\), i.e., lower investments lead to further PPS divergence based purely on risk premium arguments.
We now turn to non-scalable investments. Lumpiness in project investments will amplify the role of strategic complementarity, with drastic consequences for the ranking of the organizational modes.

4 Lumpy Investments

Suppose now that investments are of fixed size, normalized so that $K = \{0, 1\}^2$, that is, each investment can either be undertaken or not. Examples are the replacement of existing equipment, M&A, or the decision to develop a new product or to enter a new market. We continue to assume that each investment is equally productive, with marginal gross return normalized to one, at fixed cost of $F(k_i) = \frac{1}{2} k_i^2$. To avoid clutter, denote the fixed cost per unit of investment by $\phi \equiv \frac{1}{2}$. The principal’s objective remains to maximize $W(k, \beta)$, as in (5), now with $K = \{0, 1\}^2$, resulting in total fixed costs of 0, $\phi$, or $2 \phi$.

4.1 Investment Incentives for Given PPS

It is useful to begin the analysis of lumpy investments by studying the Date-2 investment subgame for given PPS and only then endogenizing the PPS. In each of these steps, we first characterize the benchmark outcome and then the outcome under the delegation regimes.

We begin by taking the PPS as given. The conditionally optimal investments the principal would choose in the contractible benchmark case, for given PPS, are $k^* = \arg\max_{k \in \{0, 1\}^2} W(k, \beta)$. Because of the investment complementarity (Lemma 1), the benchmark investment profile will be “all or nothing:” for a fixed cost threshold $\phi^*(\beta)$, the principal undertakes both investments ($k^* = (1, 1)$) if $\phi \leq \phi^*(\beta)$, but neither of them ($k^* = (0, 0)$) if $\phi > \phi^*(\beta)$.\(^{20}\) We refer to $\phi^*(\beta)$ as the benchmark fixed cost threshold, for given $\beta$.

\(^{20}\)It is easy to see that the principal would be indifferent between the “mixed” investment profiles $(1, 0)$ and $(0, 1)$. However, by virtue of investment complementarity (Lemma 1), such a mixed investment profile can never be optimal.
Now turn to delegated investment decisions when the latter are lumpy and noncontractible. Strategic complementarity affects the set of investment profiles that can arise in equilibrium. Several properties carry over directly from the scalable investment setting:

P1: Manager $i$’s gross investment return $\Gamma(\cdot)$ displays strategic complementarity in investments for given PPS (Lemma 1).

P2: Higher PPS lowers a manager’s investment incentives because of the investment-risk link (Lemmas 2 and 3).

P3: For given PPS, any delegation regime results in underinvestment due to hold-up (Lemmas 2 and 3). Thus the principal will choose the regime that provides the manager(s) with maximum investment incentives in aggregate.

For the sake of illustration, and with slight abuse of notation, for now denote the PPS profile by the non-ordered pair $\beta = (\underline{\beta}, \bar{\beta})$, where $\underline{\beta} < \bar{\beta}$.

Under bundling, Manager $\ell$’s optimization problem remains as in (9), with investments now chosen from the discrete set $\mathcal{K} = \{0, 1\}^2$ at fixed cost $\phi(k_A + k_B)$. By the same logic as for scalable investments (and by P2 and P3, above), if the principal decides to bundle investment decision rights, she will do so in the hands of the manager with lower-powered incentives, $\underline{\beta}$. By P1, the equilibrium investment profile will again be all-or-nothing: the investment center manager under bundling invests $k^\ell(\underline{\beta}) = (1, 1)$, for given PPS, if

$$\Gamma(1, 1 | \underline{\beta}) - 2\underline{\beta}\phi \geq \Gamma(0, 0 | \underline{\beta}),$$

and $(0, 0)$ otherwise. Denote by $\phi^{\ell}_{11}(\underline{\beta})$ the fixed cost threshold at which this investment condition becomes binding.

Under the split regime, a Nash equilibrium in investments, $k^S_i(\beta)$, is determined by (8), now with $k_i \in \{0, 1\}$ at fixed cost $F(k_i) = \phi k_i$. By P2, the

\[21\] It is straightforward to show that, given (6), the fixed cost threshold below which both managers invest for given contracts is lower than that in the contractible benchmark.
investment bottleneck is the manager with the greater PPS. Hence, the investment profile \( k^S(\beta) = (1, 1) \) constitutes an equilibrium for fixed costs low enough such that even the high-PPS manager has no incentive to deviate:

\[
\Gamma(1, 1 | \beta) - \beta \phi \geq \Gamma(1, 0 | \beta).
\]

(12)

At the same time, \( k^S(\beta) = (0, 0) \) is an equilibrium for \( \phi \) high enough such that even the low-PPS manager has no incentive to invest unilaterally:

\[
\Gamma(1, 0 | \beta) - \beta \phi \leq \Gamma(0, 0 | \beta).
\]

(13)

Denote by \( \phi_{11}(\beta) \) the fixed cost threshold at which (12) becomes binding and by \( \phi_{00}(\beta) \) that at which (13) becomes binding. Clearly, the unique investment equilibrium is \((1, 1)\) for sufficiently low fixed costs and \((0, 0)\) for high fixed costs.

For intermediate fixed costs, the nature of equilibrium depends on the ranking of the \( \phi \)-thresholds, which in turn will depend on the managers’ PPS differential:

Lemma 4 The (Pareto-dominant) investment equilibrium under the split regime, given lumpy investments \((k \in \{0, 1\}^2)\) and \(\beta = (\underline{\beta}, \overline{\beta})\), is:

(a) For \(\overline{\beta} - \underline{\beta} \leq \frac{2 - \rho \eta \beta}{\rho n (\mu + 1)}\), \(\phi_{00}(\beta) \leq \phi_{11}(\beta)\) and \(k^S(\beta) = \begin{cases} (1, 1), & \text{if } \phi \leq \phi_{11}(\beta), \\ (0, 0), & \text{if } \phi > \phi_{11}(\beta). \end{cases}\)

(b) For \(\overline{\beta} - \underline{\beta} > \frac{2 - \rho \eta \beta}{\rho n (\mu + 1)}\), \(\phi_{00}(\beta) > \phi_{11}(\beta)\) and \(k^S(\beta) = \begin{cases} (1, 1), & \text{if } \phi \leq \phi_{11}(\beta), \\ (1, 0) \text{ or } (0, 1), & \text{if } \phi \in (\phi_{11}(\beta), \phi_{00}(\beta)], \\ (0, 0), & \text{if } \phi > \phi_{00}(\beta). \end{cases}\)

Games of strategic complementarity routinely have multiple equilibria; this is true also for the split regime if managers face comparable PPS (Lemma 4a).

For intermediate fixed costs values, \((1, 1)\) and \((0, 0)\) each are Nash equilibria, because (12) and (13) hold simultaneously, but the invest-invest equilibrium is
Figure 3: Equilibrium investments under the split regime for given PPS, $\beta = (\underline{\beta}, \overline{\beta})$

the Pareto-dominant one here.\footnote{This holds generally for supermodular games such as ours (Milgrom and Roberts, 1990, Theorem 7). The Pareto-dominant (1, 1) equilibrium is also the one preferred by the principal.} For $\phi \in (\phi_{00}(\beta), \phi_{11}(\beta))$, hence, we can ignore

the no-investment equilibrium (Figure 3a).

Lemma 4b states a condition for an asymmetric equilibrium to obtain for intermediate fixed cost values. In that equilibrium, only the manager with lower-powered incentives will invest. While his investment raises the investment incentive also for the other manager, if the PPS differential is large enough, then this strategic complementarity effect is insufficient to compensate the high-PPS manager for the incremental risk premium associated with investing (Figure 3b).

We first approach the principal’s decision problem for lumpy investments heuristically by asking which regime implements the investment profile (1, 1) for higher fixed cost parameters. This amounts to comparing the $\phi$-thresholds defined by (11) and (12), for given $\beta = (\underline{\beta}, \overline{\beta})$. Conditional on bundling decision rights, to boost investments, the principal again would designate the manager with the more muted incentives the investment center manager. Denote by $\ell^*$ the manager facing the lower PPS, $\underline{\beta}$. We say the split regime investment-dominates bundling (for given PPS) whenever $\phi_{11}(\beta) \geq \phi_{11}^*(\underline{\beta})$, and vice versa.\footnote{By P3, both these thresholds are smaller than the benchmark threshold $\phi^*(k)$. On a technical level, we use here the fact that $W(\beta, k)$ is concave in $k$ for given $\beta.$}
bundling; (ii) bundling entails a risk tolerance benefit because it allows the principal to assign investment authority to the manager with a lower PPS. Our next result identifies conditions to evaluate this tradeoff:

**Lemma 5** Given lumpy investments \( k \in \{0, 1\}^2 \) and PPS of \( \beta = (\overline{\beta}, \beta) \):

(a) For \( \overline{\beta} - \beta < \frac{1 - \rho\eta \beta}{\rho(\mu + \frac{1}{2})} \), the split regime investment-dominates bundling.  

(b) For \( \overline{\beta} - \beta > \frac{3/2 - \rho\eta \beta}{\rho(\mu + \frac{1}{2})} \), bundling with \( \ell^* \) investment-dominates the split regime.

This result is in stark contrast to Proposition 1 for scalable investments. Why, with lumpy investments, does the split regime generate stronger investment incentives if the PPS differential is limited (Lemma 5a)? As \( \overline{\beta} - \beta \) becomes small, the risk tolerance benefit of bundling vanishes—this leaves the different game forms. Consider the limit case where both PPS levels converge to the same value, say \( x \). Comparing (11) with (12), we find that inducing \((1, 1)\) as a Nash equilibrium (split regime) is a less demanding condition than inducing some Manager \( \ell \) to invest two units rather than none (bundling). The reason is that, for any given \( x \),

\[
\Gamma(1, 1 \mid \beta_i = x) - \Gamma(1, 0 \mid \beta_i = x) - 2 \Gamma(1, 1 \mid \beta_\ell = x) - \Gamma(0, 0 \mid \beta_\ell = x) > 0,
\]

by strategic complementarity (Lemma 2). The split regime generates strong investment incentives in aggregate by requiring that investing only be each manager’s best response to the other manager investing. At an internalized fixed cost of \( \beta_i \phi \), Manager \( i \) reaps his share of the return from changing the investment profile from \((0, 1)\) to \((1, 1)\). Investment incentives under bundling, in contrast, are muted by the fact that the investment center manager has to pay for the total fixed cost, \( 2\beta_\ell \phi \), to change the investment profile from \((0, 0)\) to \((1, 1)\).

With project proceeds split by the managers, eliciting high levels of inputs from two players in form of a Nash equilibrium is “cheap” if these inputs are
(a) Bundling: low-PPS manager chooses $k_A$ and $k_B$

(b) Split regime: Nash equilibrium (binding constraint is high-PPS manager)

**Figure 4:** Comparison of investment incentives with exogenous PPS
Solid arrows indicate the binding constraints in order to elicit the investment profile $(1, 1)$.

We henceforth label the game form effect the *strategic complementarity effect*: the split regime takes advantage of strategic complementarity; bundling does not. A potential deviation from equilibrium entails lowering total investments from two to one units under the split regime—and from two to zero units under bundling. While, for scalable investments (Section 3), the relevant deviations are infinitesimally small, irrespective of the regime, lumpiness magnifies the strategic interaction in that $k_i$ scales up the marginal product of $k_j$ in a discrete manner. Figure 4 illustrates the risk tolerance and strategic complementarity effects, with bold arrows indicating the binding investment constraints under the regimes.

Now consider the case of a large PPS differential (Lemma 5b); specifically, fix $\beta$ while increasing $\overline{\beta}$. Investment incentives under bundling are unaffected by this change, because only $\beta$ then matters for investments. The bottleneck under the split regime is to get the high-PPS manager to invest; this constraint becomes tighter as $\overline{\beta}$ increases. Letting $\eta$ vary, as a measure of the intrinsic project
uncertainty, Figure 5 plots the fixed cost thresholds as $\beta$ increases. High $\eta$ values boost the induced risk tolerance effect and dampen the strategic complementarity (the increasing differences property of the risk premium in the investments, as per (7), becomes more pronounced). Both effects work in tandem to shrink the region for which the split regime is the investment-dominant one (Figure 5a).

4.2 Regime Comparison with Equilibrium Contracts

We now show that the main conclusion of the preceding subsection—on the potential benefits of splitting decision rights—extends to endogenous contracts. The optimal PPS for given investment levels under the contractible benchmark is $\beta^o(k)$, as in (10), now with $K_i = \{0, 1\}$, and the benchmark investments maximize $W^*(k) \equiv W(k, \beta^o(k))$. As shown in the proof of Lemma 6, the expected surplus continues to display investment complementarity even after endogenizing the contracts, i.e., the value function $W^*(k)$ has increasing differences. As a result, the principal’s choice of lumpy investments in the benchmark case continues to be all-or-nothing, even with endogenous contracts.

Lemma 6 For lumpy investments $(k \in \{0, 1\}^2)$, the contractible benchmark solution calls for both investments to be made $(k^* = (1, 1))$ and $\beta^* = \beta^o(1, 1)$ if
\[ \phi \leq \phi^* \equiv \phi^*(\beta^o(1,1)), \text{ and calls for } k^* = (0,0) \text{ and } \beta^* = \beta^o(0,0) \text{ otherwise.} \]

With delegated lumpy investments, the managers’ contracts again are chosen by the principal at Date 1, anticipating the induced Date-2 investment and general effort choices. This causes a technical challenge under the split regime, as the PPS profile \( \beta \) affects qualitatively the set of possible equilibria in the investment subgame (Lemma 4). To address this issue, for the remainder of this section, we impose an additional upper bound on the intrinsic project risk, \( \eta \leq \eta_{sym} \equiv \frac{2}{\rho(\mu + \frac{1}{2})} \), which is incremental to the bounds in (6) and ensures that Lemma 4 holds, for any PPS. Thus:

\[ \eta \leq \min \{ \eta_{risk}, \eta_{pos}, \eta_{sym} \}. \quad (6) \]

Hence, when optimizing over contracts, the principal only needs to consider symmetric investment equilibria, \((0,0)\) or \((1,1)\), played by the managers.

Under the split regime, if the principal were to set the PPS equal to \( \beta^o(1,1) \), then the managers would play the \((1,1)\) investment equilibrium up to a fixed cost level of \( \phi^S \equiv \phi^S(\beta^o(1,1)) \). At the same time, beyond fixed costs of \( \phi^* \) investments are lost—and the PPS adjusted to \( \beta^o(0,0) \)—even in the benchmark case (Lemma 5); a fortiori, the same holds under the split regime. Hence, for fixed cost values that are either very low or very high, incomplete contracting does not impose any additional cost on the principal under the split regime. However, to induce the investment profile \((1,1)\) for intermediate fixed cost values, \( \phi \in (\phi^S, \phi^*) \), the principal must lower the PPS, so that the investment incentive condition,

\[ \Gamma(1,1 \mid \beta_i) - \beta_i \phi \geq \Gamma(1,0 \mid \beta_i) \quad (15) \]

holds for each Manager \( i \). Denote by \( \tilde{\beta}^S(\phi) \) the PPS that satisfies this condition as an equality.

The optimal contracts and equilibrium investment outcome under the split regime are described as follows:
Proposition 2 (Split regime) Suppose investments are lumpy ($k \in \{0, 1\}^2$), and \( [\theta] \) holds. Under the split regime, there exists a unique threshold $\phi^S \in (\bar{\phi}, \phi^*)$, such that:

(a) For $\phi \in (\bar{\phi}, \phi^S]$, the principal induces both investments, \( k = (1, 1) \) (as in the benchmark), by lowering the PPS to $\beta_B = \max\{\bar{\beta}^S(\phi), 0\} < \beta^o_B(1, 1)$ and $\beta_A = \min\{\beta^o_A(1, 1), \max\{\bar{\beta}^S(\phi), 0\}\}$. 

(b) For $\phi \in (\phi^S, \phi^*)$, the principal foregoes investments altogether, \( k = (0, 0) \), by raising both PPS to $\beta_i = \beta^o_i(0, 0)$, above the benchmark level of $\beta^o_i(1, 1)$.

Figure 6a illustrates this result graphically for increasing fixed costs. Manager B has a higher benchmark PPS, because he faces a less volatile environment. As fixed costs increase, his incentives have to be muted first, to $\beta_B = \bar{\beta}^S(\phi)$, to elicit his investment.\(^\text{24}\) As fixed costs grow further, at some point, the investment constraint \(^\text{15}\) may become binding even for Manager A; if so, both managers’ incentives must be muted in lockstep. At fixed cost of $\phi^S$, the return to the investments falls short of the opportunity cost of foregone general effort, so the principal gives up on investments. This, in turn, allows her to expose both managers to higher-powered incentives because of the reduced project risk.

Under bundling, recall that the principal chooses $\ell = A$ to take advantage of the induced risk tolerance effect. Incomplete contracting imposes costs on the principal again only for intermediate fixed cost levels, $\phi \in (\bar{\phi}^{\ell = A}, \phi^*)$, where $\bar{\phi}^{\ell = A} \equiv \phi^{\ell = A}_{11}(\beta^o(1, 1))$. Manager A chooses $(1, 1)$ under bundling if and only if

$$\Gamma(1, 1 \mid \beta_A) - 2\beta_A \phi \geq \Gamma(0, 0 \mid \beta_A). \quad (16)$$

Denote by $\bar{\beta}^{\ell = A}(\phi)$ the PPS level that satisfies this condition as an equality. By strategic complementarity, it is immediate that $\bar{\beta}^{\ell = A}(\phi) < \bar{\beta}^S(\phi)$, for any $\phi$. The optimal contracts and equilibrium investment outcome under bundling are:

\(^{24}\)If $\bar{\beta}^S(\phi) < 0$, the principal sets the PPS to zero, which makes the investing manager indifferent and therefore he invests according to the principal’s preference.
Proposition 3 (Bundling) Suppose investments are lumpy \((k \in \{0, 1\}^2)\) and \(\phi \in \{0, 1\}\) holds. Under bundling, Manager A, who faces high general uncertainty, is the designated investment center manager \((\ell = A)\), and there exists a unique threshold \(\phi^{\ell=A} \in (\bar{\phi}^{\ell=A}, \phi^*)\), such that:

(a) For \(\phi \in (\bar{\phi}^{\ell=A}, \phi^{\ell=A})\), the principal induces both investments, \(k = (1, 1)\) (as in the benchmark), by lowering Manager A’s PPS to \(\beta_A = \max\{\beta^{\ell=A}(\phi), 0\} < \beta_A^0(1, 1)\), while keeping that for Manager B at \(\beta_B = \beta_B^0(1, 1)\).

(b) For \(\phi \in (\phi^{\ell=A}, \phi^*)\), the principal foregoes investments altogether, \(k = (0, 0)\), by raising both PPS to \(\beta_i = \beta_i^0(0, 0)\), above the benchmark level of \(\beta_i^0(1, 1)\).

Propositions 2 and 3 have in common that for intermediate fixed cost levels, the principal trades off distortions in project investments and general efforts. Yet, the nature of the contract adjustments necessary to elicit investments differs qualitatively across the regimes (Figure 6). Recall that for scalable investments, incomplete contracting calls for bundling, which in turn is associated with PPS divergence in equilibrium (see end of Section 3). This PPS divergence result un-
der bundling carries over to lumpy investments. Under the split regime, in contrast, incomplete contracting leads to PPS convergence across divisions whenever the investment constraints are binding: the high-PPS manager is the investment bottleneck—his incentives need to be muted (first). Throughout the paper, we assume that decision rights can be moved across divisions at no direct cost. This may not always be the case. For firms that are stuck with the split regime for technological reasons, our model predicts harmonized incentives.

Having characterized the optimal contractual adjustments and attendant equilibrium investments under the two regimes, we are now in a position to generalize the result of Lemma 5 regarding the optimal regime choice.

**Proposition 4** Suppose investments are lumpy \((k \in \{0, 1\}^2)\), and \((6'\) holds.

(a) The principal prefers the split regime if the managers face sufficiently similar levels of general uncertainty \((\sigma_A^2 - \sigma_B^2\) is sufficiently small).

(b) The principal prefers bundling (with Manager A as investment center manager) if the managers face sufficiently different levels of general uncertainty \((\sigma_A^2 - \sigma_B^2\) is sufficiently large) and the cost of effort, \(v\), is sufficiently low.

The message from the heuristic analysis for given contracts generalizes to endogenous (linear) contracts. For lumpy specific investments, the principal trades off the strategic complementarity benefits of the split regime and the greater induced risk tolerance, associated with low-powered PPS, under bundling. As the general uncertainty levels converge across divisions, the risk tolerance benefit becomes negligible, and the split regime is preferred (Fig.7a,b). Bundling all investment authority in the hands of one manager and muting his incentives would boost both investments (the two-for-one effect). But because it fails to take full advantage of the investment complementarity, the PPS reduction required to elicit the investments would be very costly in terms of foregone general effort exerted by Manager \(\ell\). Only if the uncertainty differential across the divisions is large (Proposition 4b and Fig.7c,d), bundling becomes the preferred mode.
Figure 7: Regime comparison with lumpy project-specific investments
Numerical example with \( \mu = 14, \rho = 2, v = 0.0001, \sigma_A^2 = 23, \eta = 0.06 \). Solid line represents the principal’s payoff under the split regime. Dashed line represents the principal’s payoff under bundling.

In connection with Proposition 1 for scalable investments, we have argued that in a pure hold-up model (where \( \rho \to 0 \)), absent the investment-risk link, the regime choice would be vacuous. This logic does not carry over to lumpy investments: the strategic complementarity effect would remain (as the sole differential force); thus the split regime would always be preferred.\(^{25}\) \(^{26}\)  

To see this, note that, as \( \rho \to 0 \), we would obtain fixed cost thresholds of \( \phi_{11} = \frac{1}{4}E_\theta[M((1, 1), \theta) - M((1, 0), \theta)] = \mu/2 + 3/4 \) under the split regime; \( \phi_{11}^\ell = \frac{1}{4}E_\theta[M((1, 1), \theta) - M((0, 0), \theta)] = \mu/2 + 1/2 \) under bundling, regardless of \( \ell = A, B \); and \( \phi^* = \frac{1}{2}E_\theta[M((1, 1), \theta) - M((0, 0), \theta)] = \mu + 1 \) for the contractible benchmark.\(^{26}\)

The continuous investment model can be recouped as a limit case of a generalized lumpy
5 Personally Costly Project Efforts

As an extension, we now consider the case where the relationship-specific inputs are not paid for with divisional funds but instead are personally costly to the respective manager who chooses them. Examples include foregone perquisites or private benefits from pet projects, or simply the disutility of engaging in time-consuming market research. This scenario may also be descriptive of startups or knowledge-based firms where the most common input is skill acquisition. Let $e_i \in \mathcal{E}_i$ denote the personally costly relationship-specific efforts—in short, “project efforts”—chosen at Date 2 at personal disutility of $G(e_i) = \frac{g}{2}e_i^2$, $g > 6$, incurred by the manager exerting it. The project effort may again be scalable ($\mathcal{E}_i = [0, \bar{e}]$) or lumpy ($\mathcal{E}_i = \{0, 1\}$). Let $e = (e_A, e_B)$.

To facilitate comparison with the earlier model section, we keep modifications to the setup to a minimum and make changes only where necessary. All functions unrelated to the cost of the project input are unchanged, i.e., $q^e(\theta, e)$, $M(\theta, e)$, $\varphi(e)$, $\beta^\rho(e)$, $\Gamma(e | \beta_i)$, and $W(e, \beta)$ remain as above except the argument $e_i$ (or $e$) replaces $k_i$ (or $k$). The maintained assumption in (6) continues to apply, with $g$ replacing $f$. The performance measures now read $\pi_i = a_i + \varepsilon_i + \frac{M(\theta, e)}{2}$, regardless of the regime (split or bundling), as the project effort cost is borne privately by the managers. The manager’s expected utility is, accordingly,

$$EU_i = \alpha_i + \beta_i \left(a_i + \frac{E[M(\theta, e)]}{2}\right) - \widetilde{FC}_i(e) - \frac{v}{2}a_i^2 - \frac{\rho}{2}\beta_i^2 \left(\sigma_i^2 + \frac{\varphi(e)}{4}\right). \quad (17)$$

Manager $i$’s effort cost is: $\widetilde{FC}_i(e) = G(e_i)$ under the split regime (each Manager $i$ bears the disutility for his project effort $e_i$), and $\widetilde{FC}_i(e) = (G(e_A) + G(e_B)) \times \mathbb{1}_{i=\ell}$ under bundling with $\ell = i$ (Manager $\ell$ bears the entire disutility of project efforts). By moving from monetary to personally costly project inputs, the managers’ general effort incentive constraints in (6) are unaffected, but their choice investment model. Suppose the investment $k_i$ is chosen from the discrete, non-binary set, $\{k_0, k_1, ..., k_n\}$, where $k_i = k_{i-1} + z$. (Our lumpy investment model has $n = z = 1$.) To approximate the continuous investment model, let $z$ become small and $n$ large; then bundling would always be optimal.
of project-specific inputs is affected in that the input cost, \( \hat{\mathcal{C}}_i(\cdot) \), is no longer scaled by the PPS (contrast [17] with [2]).

In the benchmark case of contractible project efforts, the principal chooses \((e^*, \beta^*)\) to maximize her expected surplus \(W(e, \beta)\). Now consider the managers’ decisions at Date 2 for noncontractible project efforts. Under the split regime, the managers choose project efforts simultaneously, such that, for given \(\beta\),

\[
\max_{e_i} \Gamma(e_i, e_j | \beta_i) - G(e_i), \quad i, j = A, B, \quad i \neq j. \tag{18}
\]

As before, we focus on pure-strategy Nash equilibria. The project efforts resulting for given \(\beta\) are denoted by \(e^S(\beta)\). Under bundling, Manager \(\ell\) solves

\[
\max_e \Gamma(e | \beta_\ell) - \sum_{m=A,B} G(e_m) \times 1_{i=\ell}, \tag{19}
\]

which yields the solution \(e^\ell(\beta)\), assumed interior. At Date 1, for any regime \(j \in \{S, \ell\}\), the principal maximizes \(W(\beta) \equiv W(e^j(\beta), \beta)\) over \(\beta\). We assume interior solutions and denote them by \(\beta^j = (\beta^j_A, \beta^j_B)\), resulting in equilibrium project efforts of \(e^j \equiv (e^j_A, e^j_B) \equiv e^j(\beta^j)\).

Consider scalable project efforts first, \(e_i \in [0, \bar{k}]\). The complementarity in specific inputs, as per Lemma 1, carries over. So does the feature that specific inputs will be underprovided in equilibrium, because of surplus division. The principal’s optimal regime choice for scalable project efforts is as follows:

**Proposition 5** For scalable project efforts \((e \in [0, \bar{e}]^2)\), bundling with \(\ell = B\) (the manager facing low uncertainty) outperforms both bundling with \(\ell = A\) and the split regime.

**Corollary 2** With scalable project efforts \((e \in [0, \bar{e}]^2)\), in equilibrium both managers face higher-powered PPS, as compared with the contractible benchmark.

The main difference to the case of monetary project inputs (Proposition 1) is that personally costly project efforts are increasing in the managers’ PPS.\footnote{In analogy with Lemmas 2a and 3a, the proof of Proposition 5 derives closed-form expressions for the project efforts resulting for given \(\beta\) under the two regimes.}
The classic moral hazard argument that stronger PPS elicits greater (project) effort is merely weakened, but not overturned, by the fact that the risk premium is again increasing in $e_i$, as $\varphi(e) = (q^*(\mu, e))^2 \eta$, for any $i$. Put differently, the first moment-effect of an increase in the PPS now dominates the second moment-effect (the input-risk link). When designing incentive contracts, the principal no longer has to trade off general effort and project-specific inputs\(^{28}\).

Bundling decision rights most effectively mitigates the underprovision of scalable project efforts. But, following the preceding arguments, now the manager facing the more stable operating environment should be designated investment center manager. A stable environment calls for high-powered PPS for Manager B, which delivers greater general-purpose and project-specific effort levels, in tandem. Lastly, because (i) equilibrium underprovision of project efforts implies lower project risk and (ii) project efforts are stimulated by greater PPS, delegation calls for uniformly higher-powered incentives as compared with the contractible benchmark outcome (Corollary\(^2\)).

Now consider lumpy project efforts, $e_i \in \{0, 1\}$. Denote the disutility incurred by a manager per unit of project effort by $\gamma \equiv \frac{g}{2}$. The principal’s objective remains to maximize $W(\beta, e)$. Solving this program, by input complementarity (Lemma\(^1\) still applies, with suitably adjusted notation), optimal project efforts are $e^* = (1, 1)$ for any effort cost below a threshold $\gamma^*$, and $e = (0, 0)$ for $\gamma > \gamma^*$. With noncontractible project efforts, following arguments familiar by now, both delegation regimes result in underprovision of project efforts, but the PPS stimulates project efforts. Under bundling, the principal would assign the decision rights over both efforts to Manager B, i.e., $\ell = B$ (as in Proposition\(^5\)). Under the split regime, the manager with the lower PPS now is the bottleneck in terms of eliciting project efforts. In equilibrium, this will be Manager A who faces greater general uncertainty. As in Section\(^4.1\) one can show (see proof of Proposition\(^6\)) that only symmetric project effort equilibria exist under the

\(^{28}\)Project efforts thus yield a version of a multitasking moral hazard model, e.g., Holmstrom and Milgrom (1990), Zhang (2003), Hughes et al. (2005).
split regime provided the managers face sufficiently similar PPS, with a sufficient condition being that
\[
\frac{\beta_{MH}^B - \beta_A^M}{\beta_B^M} \leq \frac{2 - \rho \eta}{2\mu + 3}.
\] (20)

This condition is met if the divisions face sufficiently similar levels of uncertainty, i.e., \((\sigma_A^2 - \sigma_B^2)\) is small, and \(\eta\) is small\(^{29}\).

The optimal regime choice is qualitatively similar to Proposition 4.

**Proposition 6** Given project efforts are lumpy \((e \in \{0, 1\}^2)\) and (20) holds:

(a) The principal prefers the split regime if the managers face sufficiently similar levels of general uncertainty, that is, if \(\sigma_A^2 - \sigma_B^2\) is small.

(b) The principal prefers bundling (with Manager B as investment center manager) if the managers face sufficiently different levels of general uncertainty, that is, if \(\sigma_A^2 - \sigma_B^2\) is large.

Under either regime, any contract adjustments to stimulate project efforts take the shape of strengthening incentives for one or both of the managers, as illustrated in Figure 8. Under the split regime (Fig.8a), the project effort constraint that ensures both managers exert their respective project efforts is a Nash equilibrium becomes binding first for Manager A (at some fixed cost level \(\gamma^S\)), so his PPS needs to be raised. At fixed costs of \(\gamma^S\), the principal gives up on project efforts—as the risk premium becomes too high—and lowers the managers’ PPS to \(\beta_S(0, 0)\). Under bundling (Fig.8b), Manager B is assigned

\(^{29}\)In the proof of Proposition 6 we show that if \((\beta - \beta)/\beta \leq \frac{2 - \rho \eta}{2\mu + 3}\), only symmetric equilibria exist under the split regime. In equilibrium, \(\beta_A^S \leq \beta_B^S\) and the relative PPS differential, \((\beta_B^S - \beta_A^S)/\beta_B^S\) is bounded from above by \((\beta^{MH}_B - \beta^{MH}_A)/\beta_B^{MH}\). To see why, note that for sufficiently low \(\gamma\), both managers invest even at \(\beta^S = \beta^S(1, 1)\); for sufficiently high \(\gamma\), even the principal prefers \((0, 0)\), and so \(\beta^S = \beta^S(0, 0)\). Inducing (1,1) for intermediate values of \(\gamma\) requires strengthening incentives—first to the low-PPS “bottleneck” Manager A. This results in convergence of the PPS for intermediate \(\gamma\). Therefore, \((\beta_B^S - \beta_A^S)/\beta_B^S \leq (\beta_B^S(0, 0) - \beta_A^S(0, 0))/\beta_B^S(0, 0) \leq (\beta^{MH}_B - \beta^{MH}_A)/\beta_B^{MH}\). Note that the sufficient condition to rule out asymmetric equilibria for monetary investments in Section 4.1 (e.g., in Lemma 4) was an upper bound on the absolute PPS differential, whereas (20) bounds the relative differential. This is again a consequence of the fact that input costs are scaled by the PPS for monetary investments but not for project efforts.
decision rights over both project efforts. At fixed costs of $\gamma^B$, his project effort constraint, which ensures he prefers $e = (1, 1)$ to $(0, 0)$, becomes binding. The principal has to raise $\beta_B$ to elicit the project efforts, which she does up to a fixed cost level of $\gamma^B$.

The tradeoff that governs the principal’s regime choice at Date 0 mirrors that for Proposition 4, except for the allocation of decision rights conditional on bundling: assigning decision rights to the manager with a more stable environment takes advantage of the moral hazard logic that higher PPS elicits greater effort, general as well as project-specific. On the other hand, the split regime again better utilizes the strategic complementarity; see Figure 9.

### 6 Conclusion Remarks

This paper derives novel predictions for the optimal assignment of decision rights across business unit managers in multidivisional firms that exhibit synergies. For scalable project-specific investments, decision rights should be bundled in the hands of one division manager—specifically, the manager who faces more volatile
operations, because his incentives will be muted, shielding him from the incremental investment-induced project risk. For scalable personally costly project efforts, which tend to play a bigger role in startups or knowledge-based firms, decision rights should again be bundled—but now in the hands of the division manager who faces more stable operations, as project efforts are stimulated by high-powered PPS. In contrast, for lumpy project-specific investments (or project efforts)—for example, capital replacement decisions (or the disutility associated with learning a programming language or getting certified as CPA/CFA)—it may be optimal to split decisions among the managers. In doing so, the principal sets

**Figure 9:** Regime comparison with (lumpy) personally costly efforts

Numerical example with $\mu = 1, \rho = 2, v = 0.1, \sigma_B^2 = 20, \eta = 0.25$. Solid line represents the principal’s payoff under the split regime. Dashed line represents the principal’s payoff under bundling.
up a non-cooperative game between the managers, so as to leverage the strategic 
complementarity that is endemic to specific inputs to joint projects.

Our model sheds light on the association of responsibility structure and man-
agers’ relative incentive strength: bundling of decision rights results in incentive 
divergence across divisions; the split regime results in incentive convergence. To 
test our predictions, it would be useful to adapt earlier empirical studies on incen-
tives and organizational processes, e.g., Nagar (2002), by distinguishing between 
the delegation of tasks that are personally costly to managers and those that call 
for managers to invest the firm’s funds in joint projects.
Appendix

Proof of Lemma 1: Given Assumption 6, it is straightforward to show that
\[
\frac{\partial^2 W(k, \beta)}{\partial k_A \partial k_B} = 1 - \frac{\rho \eta}{4} (\beta_A^2 + \beta_B^2) \geq 1 - \frac{\rho \eta}{2} > 0 \quad \text{and} \quad \frac{\partial^2 \Gamma(k|\beta)}{\partial k_A k_B} = \beta_i \left( \frac{1}{2} - \frac{\rho \eta}{4} \beta_i \right) \propto 1 - \frac{\rho \eta}{2} \beta_i \geq 1 - \frac{\rho \eta}{2} > 0, \quad i = A, B.
\]

Proof of Lemma 2:

Part (a): For given PPS, the best response for Manager \(i\) to an anticipated investment \(k_j\) by his counterpart, \(k_i(k_j \mid \beta_i)\), is found by setting the first-order conditions corresponding to (8) equal to zero; upon rearranging:
\[
k_i(k_j \mid \beta_i) = \frac{\mu + k_j}{f + \frac{\rho \eta}{2} \beta_i - \frac{1}{2}}, \quad i = A, B, \quad j \neq i.
\]
Because \(k_i(k_j = 0 \mid \beta_i) > 0\) and \(\frac{d}{dk_j} k_i(k_j \mid \beta_i) < 1\) for any \(k_j, \beta, i\) and \(j \neq i\), we have a unique Nash equilibrium in investments for any PPS. (To bound the slope of the reaction curves by one, we use the maintained assumption that \(f > 6\).) Solving for the equilibrium in closed form:
\[
k_i^S(\beta) = \frac{\mu (2 - \rho \eta \beta_i)}{\rho \eta (\beta_i + \beta_j) + 4(f - 1)}, \quad i = A, B, \quad j \neq i. \tag{21}
\]
The equilibrium investments are decreasing in either manager’s PPS:
\[
\frac{dk_i^S(\beta)}{d\beta_i} = \frac{-\mu \rho \eta (\rho \eta \beta_j + 4f - 3)}{(\rho \eta (\beta_i + \beta_j) + 4(f - 1))^2} < \frac{dk_i^S(\beta)}{d\beta_j} = \frac{-\mu \rho \eta (1 - \rho \eta \beta_i)}{(\rho \eta (\beta_i + \beta_j) + 4(f - 1))^2} < 0. \tag{22}
\]

Part (b): We first show that \(k_i^* > k_i^S, \quad i = A, B\). Recall that under the benchmark case the principal contracts on \(k_A^* = k_B^* = k^*\). We fix \(k_j\) at \(k_j = k^*\) and consider the best response of Manager \(i\), described by the first-order condition that corresponds to (8):
\[
y_i^S(k_i, k_j^*) \equiv q^*(\mu, k_i, k_j^*) \left( \frac{1}{2} - \frac{\rho \eta}{4} \beta_i \right) - f k_i = 0.
\]
Contrast that with the conditionally optimal choice of contractible (benchmark) investment from the viewpoint of the principal, found by taking the derivative
of $W(k, \mathbf{\beta})$ with respect to $k_i$:

$$y^*_i(k_i, k^*_j) \equiv q^*(\mu, k_i, k^*_j) \left(1 - \frac{\rho \eta}{4} \sum_{m=A,B} (\beta_m^o(k_i, k^*_j))^2 \right) - f k_i = 0.$$ 

Now:

$$y^*_i(k_i, k^*_j) - y^S_i(k_i, k^*_j) = q^*(\mu, k_i, k^*_j) \left(1 - \frac{\rho \eta}{4} \sum_{m=A,B} (\beta_m^o(k_i, k^*_j))^2 - \frac{1}{4} + \frac{\rho \eta}{4} \beta_i \right) \propto 1 - \frac{\rho \eta}{2} \left( \sum_{m=A,B} (\beta_m^o(k_i, k^*_j))^2 - \beta_i \right) > 1 - \rho \eta \geq 0.$$ 

By strategic complementarity (Lemma 1), therefore, both managers underinvest under the split regime.

Next, we show that $k^S_A > k^S_B$. By (21), $k^S_A > k^S_B$ if and only if $\beta^S_A < \beta^S_B$. We prove by contradiction that the latter inequality holds. Suppose not, i.e., that the optimal PPS is $\beta_A = \beta_o > \beta_{oo} = \beta_B$. By (21), this would yield $k_A = k_o < k_{oo} = k_B$. It is useful to write

$$\Phi(\beta_i, \sigma_i^2) \equiv a_i(\beta_i) - \frac{\nu}{2} (a_i(\beta_i))^2 - \frac{\rho}{2} \beta_i \sigma_i^2$$

to summarize the surplus components related to the divisional moral hazard problems. The principal’s expected payoff then can be presented as:

$$W((\beta_o, \beta_{oo}), (k_o, k_{oo})) = \Phi(\beta_o, \sigma_o^2) + \Phi(\beta_{oo}, \sigma_{oo}^2) + E[M(\theta, (k_o, k_{oo}))] - F(k_o) - F(k_{oo}) - \frac{\rho}{2} \sum_i \beta_i^2 \varphi(k_o, k_{oo}).$$

Now suppose that the principal “swaps” the PPS of the managers, i.e., $\beta_A = \beta_{oo} < \beta_o = \beta_B$. This will also swap the investments, i.e., $k_A = k_{oo} > k_o = k_B$. Given the assumed identical productivity and investment fixed cost functions, the expected surplus $E[M(\theta, (k_{oo}, k_o))]$, the surplus variance $\varphi(k_{oo}, k_o)$ and the
Because \( \Phi(\beta_i, \sigma_i^2) \) has decreasing differences in \( \beta_i \) and \( \sigma_i^2 \). If \( \beta^S_i < \beta^{MH}_i \) for any \( i \), the surplus effect of the PPS swap, \( \Delta \) in (23), is positive (because \( \sigma^2_B < \sigma^2_A \)). Hence, \( k^S_A > k^S_B \) must hold in equilibrium.

**Proof of Proposition 1** In Step 1 we show that bundling with \( \ell = A \) outperforms the split regime; in Step 2, that it outperforms bundling with \( \ell = B \).

**Step 1.** Fix the PPS at \( \mathbf{\beta} = \mathbf{\beta}^S \) and consider the equilibrium under bundling with \( \ell = A \) at this PPS level. Then, \( k^\ell_{i=A}(\mathbf{\beta}^S) > k^S_i(\mathbf{\beta}^S) \), for any \( i \), by \( \beta^S_A < \beta^S_B \) and strategic complementarity (Lemma 2). For \( W^S(\mathbf{\beta}) \equiv W(k^S(\mathbf{\beta}), \mathbf{\beta}) \) and \( W^\ell(\mathbf{\beta}) \equiv W(k^\ell(\mathbf{\beta}), \mathbf{\beta}) \), it follows that \( W^S(\mathbf{\beta}^S) \leq W^\ell_{i=A}(\mathbf{\beta}^S) \), because: (i) \( W(\cdot) \) is concave in \((k_A, k_B)\) for given \( \mathbf{\beta} \), and (ii) \( k^\ell_{i=A} \leq k^\star \), for any \( i \), as per Lemma 3. By revealed preference, \( W^\ell_{i=A}(\mathbf{\beta}^S) \leq W^\ell_{i=A}(\mathbf{\beta}^\ell_{i=A}) \equiv W(k^\ell_{i=A}(\mathbf{\beta}^\ell_{i=A}), \mathbf{\beta}^\ell_{i=A}) \).

**Step 2.** Note that \( q^\star(\theta, \mathbf{k}) \) depends solely on the sum of the investments, henceforth denoted \( \kappa \equiv \sum_{i=A,B} k_i \), and so does the conditionally optimal PPS of Manager \( i \) in the benchmark solution. Therefore, slightly abusing notation, we write \( q^\star(\theta, \kappa) \) (instead of \( q^\star(\theta, \mathbf{k}) \)) and \( \beta^\star_i(\kappa) \) (instead of \( \beta^\star_i(\mathbf{k}) \)).

Denote the optimal PPS for Manager B under bundling with \( \ell = B \) by \( \beta^\ell_{i=B} = \bar{\beta} \), resulting in total units of equilibrium investment \( \bar{\kappa} \equiv \sum_{i=A,B} k^\ell_{i=B}(\bar{\beta}) \). The optimal PPS vector under bundling with \( \ell = B \) then is \( \mathbf{\beta}^\ell_{i=B} = (\beta_A^\star(\bar{\kappa}), \bar{\beta}) \). Note that \( \bar{\beta} < \beta^0_B(\bar{\kappa}) \). Now consider bundling with \( \ell = A \) and suppose the principal chooses a PPS vector of \( \mathbf{\beta} = (\beta_A = \bar{\beta}, \beta_B = \beta_B^0(\bar{\kappa})) \), resulting again in total investment of \( \sum_i k^\ell_{i=A}(\bar{\beta}) = \bar{\kappa} \). Because \( \sigma^2_A > \sigma^2_B, \beta^0_B(\bar{\kappa}) > \beta^3_A(\bar{\kappa}), \) but there are two possible cases regarding the ranking of \( \beta^0_A(\bar{\kappa}) \) and \( \bar{\beta} \).
Case 1: We claim that for $\beta_B^o(\kappa) > \beta_A^o(\kappa) > \overline{\beta}$ bundling with $\ell = A$ dominates bundling with $\ell = B$. Denoting $\chi(\beta_i | \kappa, \sigma_i^2) \equiv \Phi(\beta_i, \sigma_i^2) - \frac{\sigma_i^2}{8} \beta_i^2 (q^*(\mu, \kappa))^2$ and $W^\ell(\beta) \equiv W(k^\ell(\beta), \beta)$, we have:

$$W^{t=A}(\beta, \beta_B^o(\kappa)) - W^{t=B}(\beta_A^o(\kappa), \overline{\beta}) = \chi(\beta \beta_A^o(\kappa) | \pi, \sigma_A^2) + \chi(\beta_B^o(\kappa) | \pi, \sigma_B^2)$$

$$- \chi(\beta_A^o(\kappa) | \pi, \sigma_A^2) - \chi(\beta | \pi, \sigma_B^2) > \chi(\beta \beta_A^o(\kappa) | \pi, \sigma_A^2) + \chi(\beta_A^o(\kappa) | \pi, \sigma_B^2)$$

$$- \chi(\beta_A^o(\kappa) | \pi, \sigma_A^2) - \chi(\beta | \pi, \sigma_B^2) = - \frac{\beta^2}{2} (\sigma_A^2 - \sigma_B^2) + \frac{\beta^2}{2} (\beta_A^o(\kappa))^2 (\sigma_A^2 - \sigma_B^2)$$

$$\propto (\beta_A^o(\kappa))^2 - \overline{\beta}^2 > 0,$$

because $\beta_A^o(\kappa) \in \text{arg max}_{\beta_j} \chi(\beta_j | \kappa, \sigma_j^2)$, $j = A, B$ and $\beta_B^o(\kappa) > \beta_A^o(\kappa) > \overline{\beta}$. By revealed preference, $W^{t=A}(\beta^t=A) > W^{t=A}(\beta, \beta_B^o(\kappa))$, i.e., bundling with $\ell = A$ outperforms bundling with $\ell = B$.

Case 2: We claim that for $\beta_B^o(\kappa) > \overline{\beta} > \beta_A^o(\kappa)$ the split regime dominates bundling with $\ell = B$. Again, fix $\beta = \beta^{t=B} = (\beta_A^o(\kappa), \overline{\beta})$ at the level optimal under bundling with $\ell = B$. If the principal were to choose this PPS vector under the split regime, the managers’ investment reaction functions would be:

$$k_B^S(k_A | \beta_B = \overline{\beta}) \equiv k_B^{t=B}(k_A | \beta_B = \overline{\beta}),$$

$$k_A^S(k_B | \beta_A = \beta_A^o(\kappa)) > k_B^{t=B}(k_A | \beta_B = \overline{\beta}).$$

By strategic complementarity, $k_B^S(\beta_A^o(\kappa), \overline{\beta}) > k_B^{t=B}$. Recall that $W(\cdot)$ is concave in $(k_A, k_B)$, given $\beta$. Hence, $W^S(\beta^{t=B}) > W^{t=B}(\beta^{t=B})$ because $k_A^{t=B} < k_i^S \leq k_i^t$, $i = A, B$ (by the observation above and Lemmas 2 [3]). By revealed preference, $W^S(\beta^S) > W^S(\beta^{t=B})$, which together with Step 1 above establishes the result.

Proof of Lemma 4: Using (12) and (13), if $\phi_{11}(\beta) - \phi_{00}(\beta) < 0$, the investment profile $(1,0)$ can be a Nash equilibrium for any $\phi \in (\phi_{11}(\beta), \phi_{00}(\beta))$. Otherwise,
if \( \phi_{11}(\beta) - \phi_{00}(\beta) \geq 0 \), then for any \( \phi \in (\phi_{00}(\beta), \phi_{11}(\beta)) \), both \((1, 1)\) and \((0, 0)\) can be equilibria simultaneously. We note that:

\[
\phi_{11}(\beta) - \phi_{00}(\beta) = \frac{\Gamma(1, 1 | \beta) - \Gamma(0, 0 | \beta)}{\beta} - \frac{\Gamma(1, 0 | \beta) - \Gamma(0, 0 | \beta)}{\beta} = \frac{E[(q^*(\theta, 1, 1))^2 - (q^*(\theta, 1, 0))^2] - \frac{\rho n}{2} (q^*(\mu, 1, 1))^2 - (q^*(\mu, 1, 0))^2}{4}.
\]

If \( \beta - \beta \leq \frac{2 - \rho n}{\rho n (\mu + 2)} \), then for any \( \phi \in (\phi_{00}(\beta), \phi_{11}(\beta)) \), both \((1, 1)\) and \((0, 0)\) can be Nash equilibria simultaneously. To predict which of these equilibria the managers will play, Theorem 7 in Milgrom and Roberts (1990) applies: if multiple equilibria exist in a supermodular game (such as the managers’ Date-2 investment game) with positive spillovers (player \( i \)'s payoff is increasing in player \( j \)'s action), then the highest equilibrium—here, \((1, 1)\)—Pareto-dominates the other equilibria. That is, for \( \phi \in (\phi_{00}(\beta), \phi_{11}(\beta)) \), we ignore the shirking equilibrium.

If \( \beta - \beta > \frac{2 - \rho n}{\rho n (\mu + 2)} \), only asymmetric equilibria—\((1, 0)\) or \((0, 1)\)—exist for intermediate \( \phi \)-values.

**Proof of Lemma 5:** We first note that

\[
\phi_{11}(\beta) = \frac{\Gamma(1, 1 | \beta) - \Gamma(0, 0 | \beta)}{2\beta} = \frac{1}{2}(\mu + 1) \left(1 - \frac{\rho n}{2} \beta\right),
\]

\[
\phi_{11}(\beta) = \frac{\Gamma(1, 1 | \beta) - \Gamma(0, 1 | \beta)}{2\beta} = \frac{1}{2} \left(\mu + \frac{3}{2}\right) \left(1 - \frac{\rho n}{2} \beta\right).
\]

Then,

\[
\phi_{11}(\beta) - \phi_{11}(\beta) = \frac{1}{2} \left[\left(\mu + \frac{3}{2}\right) \left(1 - \frac{\rho n}{2} \beta\right) - (\mu + 1) \left(1 - \frac{\rho n}{2} \beta\right)\right] \propto \frac{1}{2} - \frac{\rho n}{2} \left(\beta \frac{1}{2} \left(\mu + \frac{3}{2}\right) - \beta \frac{1}{2} (\mu + 1)\right).
\]

40
The sufficient conditions in the lemma follow immediately.

**Proof of Lemma 6:** Recall that the principal’s payoff in the benchmark case can be written as $W^*(k) \equiv W(\beta^o(k), k)$, where $\beta^o(k)$ is as defined in (10). It is straightforward to see that the benchmark solution is $k^* = (1, 1)$ and $\beta^* = \beta^o(1, 1)$ for small enough $\phi$, and $k^* = (0, 0)$ and $\beta^* = \beta^o(0, 0)$ for sufficiently high $\phi$. It remains to consider the solution for intermediate values of $\phi$. To show that it is never optimal for the principal to contract on $k = (1, 0)$ and $\beta = \beta^o(1, 0)$ (or, equivalently, on $k = (1, 0)$ and $\beta = \beta^o(0, 1) = \beta^o(1, 0)$) we need to show that $\phi_{2\to1} \geq \phi_{2\to0}$, where $\phi_{2\to1} \equiv W^*(1, 1) - W^*(1, 0)$ and $\phi_{2\to0} \equiv \frac{1}{2}[W^*(1, 1) - W^*(0, 0)]$. Now: $\phi_{2\to1} - \phi_{2\to0} \approx 2(W^*(1, 1) - W^*(1, 0)) - (W^*(1, 1) - W^*(0, 0)) \geq 0$, because $W^*(1, 1) - W^*(1, 0) \geq W^*(1, 0) - W^*(0, 0)$. To see why, treat for a moment $k$ as a continuous variable and note that

$$dW^* dk = \frac{\partial W^*}{\partial k} = E[M_{kA}(\theta, k)] - \rho \sum_i (\beta_i^o(k))^2 \frac{\partial \phi(k)}{\partial k} - F'(k_A)$$

and

$$\frac{dW^*}{dk} = q(\mu, k) \left(1 - \frac{\rho \eta}{4} \sum_i (\beta_i^o(k))^2\right) - f k_A$$

$$\frac{d^2W^*}{dk_A dk_B} = \frac{\partial^2 W^*}{\partial k_A \partial k_B} = q_{k_A}(\mu, k) \left(1 - \frac{\rho \eta}{4} \sum_i (\beta_i^o(k))^2\right) - q(\mu, k) \frac{\rho \eta}{2} \sum_i \left(\beta_i^o(k) \frac{\partial^2 \beta_i^o(k)}{\partial k_B}\right).$$

In summary, the contractible benchmark solution is $(k^*, \beta^*) = ((1, 1), \beta^o(1, 1))$, if $\phi \leq \phi^* \equiv \phi^o(\beta^o(1, 1))$, and $(k^*, \beta^*) = ((0, 0), \beta^o(0, 0))$, otherwise.

**Proof of Proposition 2:** Given Condition (6'), we can ignore asymmetric investment equilibria under the split regime. Hence, the principal will choose the optimization program with the greater value from among the following:
$\mathcal{P}^S_{11}$ (Induce investment under the split regime): For any $\phi \in (\bar{\phi}^S, \phi^*)$,

$$\max_{\beta} \ W(\beta \mid k = (1,1)), \text{ subject to: } \begin{cases} 3 \end{cases} \text{ and } \beta_i \leq \bar{\beta}^S(\phi), \text{ for any } i.$$  

$\mathcal{P}^S_{00}$ (Forestall investment under the split regime): For any $\phi \in (\bar{\phi}^S, \phi^*)$,

$$\max_{\beta} \ W(\beta \mid k = (0,0)), \text{ subject to: } \begin{cases} 3 \end{cases} \text{ and } \beta_i > \bar{\beta}^S(\phi), \text{ for any } i.$$

Here, $\bar{\beta}^S(\phi)$ makes the investment constraint (15) binding. Let $\bar{\beta}^S(\phi) \equiv \max\{\bar{\beta}^S(\phi), 0\}$. In order to elicit both investments (Program $\mathcal{P}^S_{11}$), the principal optimally sets $\beta_B = \bar{\beta}^S(\phi)$ and $\beta_A = \min\{\beta_A^0(1,1), \bar{\beta}^S(\phi)\}$. (Note that when $\beta_i = 0$, the manager is indifferent and invests whenever it benefits the principal.) The no-investment program $\mathcal{P}^S_{00}$ calls for $\beta_i = \beta_i^0(0,0), \ i = A, B$ for any $\phi > \bar{\phi}^S$.

Denote by $K^*$ the value of program $\mathcal{P}^*$ and by $K^S_k$ the value of program $\mathcal{P}^S_k$, $k = (0,0), (1,1)$. For any $\phi \in (\phi^S, \phi^*)$:

$$K^*(\phi) = W(\beta^0(1,1), k = (1,1) \mid \phi),$$

$$K^S_{11}(\phi) = W(\min\{\beta_A^0(1,1), \bar{\beta}^S(\phi)\}, \bar{\beta}^S(\phi), k = (1,1) \mid \phi),$$

$$K^S_{00} = W(\beta^0(0,0), k = (0,0)).$$

Begin by considering fixed cost values $\phi = \bar{\phi}^S + \nu, \nu \to 0$. Then $K^S_{11}(\phi) = K^*(\phi) - \delta, \delta \to 0$, because $\bar{\beta}^S(\phi)$ is a continuous and decreasing function of $\phi$ and $W(\min\{\beta_A^0(1,1), \bar{\beta}^S(\phi)\}, \bar{\beta}^S(\phi), k = (1,1) \mid \phi)$ is continuous and decreasing in $\phi$ and increasing in $\beta_A$. That is, the value of program $\mathcal{P}^S_{11}$ converges to that of the benchmark program $\mathcal{P}^*$ as $\delta$ becomes small. At the same time, $K^S_{00}$ is bounded away from $K^*(\phi)$ for $\phi$ close to $\phi^S$. This holds because, adapting Lemma 2 to lumpy investments shows that $\phi^S < \phi^*$, combined with the observations that (i) $K^S_{00} = K^*(\phi = \phi^*)$ and (ii) $K^*(\phi)$ is monotonically decreasing in $\phi$. Thus, we have shown that for $\phi \downarrow \phi^S$, $K^S_{11}(\phi) > K^S_{00}$, whereas for $\phi \uparrow \phi^*$, $K^S_{11}(\phi) < K^S_{00}$.

Lastly, since $K^S_{11}(\phi)$ is monotonically decreasing in $\phi$ whereas $K^S_{00}$ is independent of $\phi$, there exists a unique indifference value $\phi^S$ at which $K^S_{11}(\phi^S) = K^S_{00}$.
Proof of Proposition 3: Given the discussion preceding the result, under bundling, the principal compares the values of the two optimization programs:

\[ P_{11}^\ell = A \] (Induce investment under bundling, \( \ell = A \)): For any \( \phi \in (\tilde{\phi}^\ell = A, \phi^*) \),

\[
\max_{\beta} W(\beta \mid k = (1, 1)), \text{ subject to: (3) and } \beta_A \leq \tilde{\beta}^\ell = A(\phi).
\]

\[ P_{00}^\ell = A \] (Foretell investment under bundling, \( \ell = A \)): For any \( \phi \in (\tilde{\phi}^\ell = A, \phi^*) \),

\[
\max_{\beta} W(\beta \mid k = (0, 0)), \text{ subject to: (3) and } \beta_A > \tilde{\beta}^\ell = A(\phi).
\]

The proof now follows similar steps as that of Proposition 2 and is available upon request.

Proof of Proposition 4:

Part (a): We first show \( \phi_S^\ell = A \leq \phi_S \). Using the proof of Lemma 5,

\[
\phi_S^\ell - \phi_S = \frac{1}{2} \left( (\mu + 1) \frac{\rho n}{2} (\beta_A^o(1, 1) - \beta_B^o(1, 1)) + \frac{1}{2} \left( 1 - \frac{\rho n}{2} \beta_B^o(1, 1) \right) \right).
\]

Fix \( \sigma_B^2 \) and note that \( \frac{\partial(\phi_S - \phi_S^\ell = A)}{\partial \sigma_A^2} = \frac{1}{2} (\mu + 1) \frac{\rho n}{2} \beta_A^o(1, 1) \beta_B^o(1, 1) < 0 \). Further, \( \lim_{\sigma_A^2 \to \sigma_B^2} (\phi_S - \phi_S^\ell = A) = \frac{1}{4} (1 - \frac{\rho n}{2} \beta_B^o(1, 1)) > 0 \). Hence, for given \( \sigma_B^2 \), there exists a threshold \( \delta_A > 0 \) such that, if \( \sigma_A^2 < \delta_A \), then \( \phi_S > \phi_S^\ell = A \). For any \( \phi \in [\tilde{\phi}^\ell = A, \phi_S] \) the principal’s payoff under bundling with \( \ell = A \) is lower than under the split regime, because under bundling she needs to reduce the investing manager’s PPS to induce \((1, 1)\), whereas under split regime the same investments are induced with the benchmark PPS.

Next, we again fix \( \sigma_B^2 \) and show that \( \phi^\ell = A \leq \phi_S \) for \( \sigma_A^2 \) sufficiently close to \( \sigma_B^2 \). Suppose not. Then, at \( \phi^\ell = A \), it must be that the payoff of the principal when she induces investment under the split regime is smaller than her payoff when she foregoes investment—that is,

\[
W((1, 1), (\beta^m(\phi^\ell = A), \tilde{\beta}^S(\phi^\ell = A)) \mid \phi^\ell = A) < W((0, 0), \beta^o(0, 0)), \quad (24)
\]
where $\beta^m(\phi) \equiv \min\{\beta^1, 1, \beta^S(\phi)\}$ and, as before, $\beta^S(\phi) \equiv \max\{\beta^S(\phi), 0\}$. Let $\tilde{\beta}_{t=A}^e(\phi) \equiv \max\{\tilde{\beta}_{t=A}^e(\phi), 0\}$. Conditional on inducing $(1, 1)$ at $\phi^{e=A}$, the principal would set the PPS at $(\tilde{\beta}_{t=A}^e(\phi^{e=A}), \beta^o_B(1, 1))$ under bundling, and at $(\beta^m(\phi^{e=A}), \beta^S(\phi^{e=A}))$ under the split regime. But as we show below, at $\phi^{e=A}$ and these PPS vectors, the principal would prefer the split regime:

$$W((1, 1), (\beta^m(\phi^{e=A}), \beta^S(\phi^{e=A})) | \phi^{e=A}) \geq W((1, 1), \tilde{\beta}_{t=A}^e(\phi^{e=A}), \beta^o_B(1, 1)) | \phi^{e=A}),$$

for $\sigma_A^2$ sufficiently close to $\sigma_B^2$. This contradicts (24), because by definition of $\phi^{e=A}$, we have $W((1, 1), (\tilde{\beta}_{t=A}^e(\phi^{e=A}), \beta^o_B(1, 1)) | \phi^{e=A}) \equiv W((0, 0), \beta^o(0, 0))$. It follows that $\phi^{e=A} \leq \phi^S$. It remains to verify that (25) holds.

Substituting the optimal general effort choice, $a_i(\beta_i) = \beta_i$, into (25) and rearranging, gives:

$$[\beta^m(\phi^{e=A}) - \tilde{\beta}_{t=A}^e(\phi^{e=A})] \left[ \frac{1}{v} - \left( \frac{\rho \sigma_A^2}{2} + \frac{\beta}{2v} + \frac{\rho}{8} \varphi(1, 1) \right) (\beta^m(\phi^{e=A}) + \tilde{\beta}_{t=A}^e(\phi^{e=A})) \right]$$

$$+ [\tilde{\beta}^S(\phi^{e=A}) - \beta^o_B(1, 1)] \left[ \frac{1}{v} - \left( \frac{\rho \sigma_B^2}{2} + \frac{\beta}{2v} + \frac{\rho}{8} \varphi(1, 1) \right) (\tilde{\beta}^S(\phi^{e=A}) + \beta^o_B(1, 1)) \right] \geq 0. \quad (26)$$

Next, we note that:

$$\beta^m(\phi^{e=A}) \geq \tilde{\beta}_{t=A}^e(\phi^{e=A}) \quad \text{and} \quad \tilde{\beta}^S(\phi^{e=A}) \leq \beta^o_B(1, 1) \quad (27)$$

because: (i) $\beta^m(\phi^{e=A}) = \min\{\beta^o_A(1, 1), \tilde{\beta}^S(\phi^{e=A})\}$, by definition; (ii) $\tilde{\beta}^S(\phi^{e=A}) \geq \tilde{\beta}_{t=A}(\phi^{e=A})$, by strategic complementarity; and (iii) $\beta^o_A(1, 1) \geq \tilde{\beta}_{t=A}(\phi^{e=A})$, as $\phi^{e=A} > \phi^o$. Hence, (26) holds if the following inequalities hold simultaneously:

$$- \left( \frac{1}{v} - \left[ \frac{\rho \sigma_A^2}{2} + \frac{\beta}{2v} + \frac{\rho}{8} \varphi(1, 1) \right] (\beta^m(\phi^{e=A}) + \tilde{\beta}_{t=A}(\phi^{e=A})) \right) \leq 0, \quad (28)$$

$$\frac{1}{v} - \left[ \frac{\rho \sigma_B^2}{2} + \frac{\beta}{2v} + \frac{\rho}{8} \varphi(1, 1) \right] (\tilde{\beta}^S(\phi^{e=A}) + \beta^o_B(1, 1)) \leq 0. \quad (29)$$

We note that (28) and (29) are equivalent to:

$$- \frac{1}{v(\beta^m(\phi^{e=A}) + \tilde{\beta}_{t=A}(\phi^{e=A}))} + \left( \frac{\rho \sigma_A^2}{2} + \frac{\beta}{2v} + \frac{\rho}{8} \varphi(1, 1) \right) \leq 0, \quad (30)$$

$$\frac{1}{v(\tilde{\beta}^S(\phi^{e=A}) + \beta^o_B(1, 1))} - \left( \frac{\rho \sigma_B^2}{2} + \frac{\beta}{2v} + \frac{\rho}{8} \varphi(1, 1) \right) \leq 0. \quad (31)$$
Summing (30) and (31) yields

\[ Q(\sigma_A^2, \sigma_B^2) = \frac{\rho}{2} (\sigma_A^2 - \sigma_B^2) - \frac{1}{\nu} \left( \frac{1}{\beta_m(\phi^A) + \beta^A(\phi^A)} - \frac{1}{\beta_S(\phi^A) + \beta^A(1, 1)} \right) \leq 0. \]  

(32)

A sufficient condition for (25) to hold is that (32) holds. Using (27) and the fact that \( \lim_{\sigma_A^2 \to \sigma_B^2} \beta^0_A(1, 1) = \beta^0_B(1, 1) \), we have

\[ \lim_{\sigma_A^2 \to \sigma_B^2} Q(\sigma_A^2, \sigma_B^2) \propto - \left( \beta^S(\phi^A) + \beta^0_B(1, 1) \right) + \left( \beta^m(\phi^A) + \beta^A(\phi^A) \right) \]

\[ \leq 2 \left( \beta^m(\phi^A) - \beta^0_B(1, 1) \right) \]

\[ = 2 \left( \beta^m(\phi^A) - \beta^0_A(1, 1) \right) \]

\[ \leq 0. \]

By continuity, it follows that (25) holds if \( \sigma_A^2 \in (\sigma_B^2, \sigma_B^2 + \delta_B) \), for small \( \delta_B > 0 \).

For any \( \phi \in [\phi^A, \phi^S] \) it follows by revealed preference that the principal’s payoff under the split regime is higher than under bundling. For any \( \phi > \phi^S \), under both regimes, the principal finds it too costly to induce \((1, 1)\) and instead induces \((0, 0)\) by setting the PPS at the benchmark level \( \beta^0(0, 0) \). Hence, for \( \phi > \phi^S \), the principal’s payoff is the same under the regimes. We cannot rank \( \phi^S \) and \( \phi^A \) unambiguously. If \( \phi^A \leq \phi^S \), no further proof is required. However, if \( \phi^A > \phi^S \), we need to show that \( W((1, 1), \hat{\beta}(\phi)) \geq W((1, 1), \hat{\beta}^A(\phi)) \) for any \( \phi \). The proof of this inequality follows similar steps as the derivation of (25) and is hence omitted.

Summarizing our results, for given \( \sigma_B^2 \), there exists a threshold \( \hat{\delta} \equiv \min\{\delta_A, \delta_B\} > 0 \) such that, if \( \sigma_A^2 - \sigma_B^2 < \hat{\delta} \), the principal prefers the split regime.

Part (b): A sufficient condition for bundling to dominate the split regime is that \( \phi^A > \phi^S \). Recall that \( \phi^A = \frac{1}{2}(\mu + 1) \left( 1 - \frac{m}{2} \beta_A^0(0, 0) \right) \) and that \( \phi^S \) is defined by \( W((1, 1), (\beta^m(\phi^S), \beta^S(\phi^S) | \phi^S) = W((0, 0), \beta^0(0, 0) | \phi^S) \). Note that \( \frac{1}{4} E[M(\theta, 1, 1) - M(\theta, 0, 0)] = \frac{1}{4} E[2(2\theta + 2)] = \mu + 1 \). Then, simplifying,

\[ \phi^S = \frac{2}{2}(\mu + 1) - \frac{m}{16} (\mu + 2)^2 (\beta^m(\phi^S))^2 + (\beta^S(\phi^S))^2 + \frac{m}{16} \mu^2 \sum (\beta^0_A(0, 0))^2 \]

\[ + \frac{1}{2} \left[ \Phi(\beta^m(\phi^S), \sigma_A^2) - \Phi(\beta_A^0(0, 0), \sigma_A^2) \right] + \frac{1}{2} \left[ \Phi(\beta^S(\phi^S), \sigma_B^2) - \Phi(\beta_B^0(0, 0), \sigma_B^2) \right] \]
and so
\[ \phi^S - \phi^x = \mu + 1 + \frac{\rho n}{2} \beta_A(0, 0)(\mu + 1) \]
\[ + \left( \Phi(\beta^S(\phi^S), \sigma_B^2) - \frac{\rho n}{8}(\mu + 2)^2(\beta^S(\phi^S))^2 \right) \equiv d_1 \]
\[ + \left( \Phi(\beta^m(\phi^S), \sigma_A^2) - \frac{\rho n}{8}(\mu + 2)^2(\beta^m(\phi^S))^2 \right) \equiv d_2 \]
\[ + \left( \Phi(\beta^o(0, 0), \sigma_A^2) - \frac{\rho n}{8}\mu^2(\beta^o(0, 0))^2 \right) \equiv d_3 \]
\[ \equiv \Delta. \]

Simplifying, we get
\[ d_1 = \frac{\tilde{\beta}^S(\phi^S)}{v} - \frac{(\tilde{\beta}^S(\phi^S))^2}{2v} - \frac{\rho}{2}(\tilde{\beta}^S(\phi^S))^2\sigma_B^2 - \frac{\rho n}{8}(\mu + 2)^2(\beta^S(\phi^S))^2 \]
\[ = \tilde{\beta}^S(\phi^S) \cdot \frac{(8 - 4\tilde{\beta}^S(\phi^S)) - 4\rho \tilde{\beta}^S(\phi^S)\sigma_B^2 - \rho \eta \mu^2(\beta^A(0, 0))}{8v} \]
\[ d_2 = \beta^o_B(0, 0) \cdot \frac{(8 - 4\beta^o_B(0, 0) - 4\rho \beta^o_B(0, 0)\sigma_B^2 - \rho \eta \mu^2(\beta^o_B(0, 0))}{8v} \]
\[ d_3 = \beta^m(\phi^S) \cdot \frac{(8 - 4\beta^m(\phi^S) - 4\rho \beta^m(\phi^S)\sigma_A^2 - \rho \eta \mu^2(\beta^m(\phi^S))}{8v} \]
\[ d_4 = \beta^o_A(0, 0) \cdot \frac{(8 - 4\beta^o_A(0, 0) - 4\rho \beta^o_A(0, 0)\sigma_A^2 - \rho \eta \mu^2(\beta^o_A(0, 0))}{8v} \]

Taking into account that \( \beta_i^o(0, 0) = \left(1 + \rho v \left(\sigma_i^2 + \frac{\mu^2}{4}\right)\right)^{-1} \), \( i = A, B \), we have
\[ d_2 = \left[2v \left(\frac{\mu^2}{4} \rho \eta + \rho \sigma_B^2 + 1\right)\right]^{-1} = \frac{1}{2v} \beta^o_B(0, 0) \text{ and } d_4 = \left[2v \left(\frac{\mu^2}{4} \rho \eta + \rho \sigma_A^2 + 1\right)\right]^{-1} = \frac{1}{2v} \beta^o_A(0, 0) \text{.} \]

Now note that for \( \sigma_A^2 \) sufficiently high, \( \beta^m(\phi^S) = \beta^o_A(1, 1) = \left[1 + \rho v \left(\sigma_A^2 + \frac{(\mu + 2)^2}{4}\eta\right)\right]^{-1} \)
and so, simplifying, \( d_3 = \left[2v \left(\frac{(\mu + 2)^2}{4} \rho \eta + \rho \sigma_A^2 + 1\right)\right]^{-1} = \frac{1}{2v} \beta^o_A(1, 1) \). Hence, \( \lim_{\sigma_A^2 \to \infty} d_3 = \lim_{\sigma_A^2 \to \infty} d_4 = 0 \) and so \( \lim_{\sigma_A^2 \to \infty} \Delta = \mu + 1 + d_1 - d_2 \equiv D \). Now note that
\[ \frac{\partial}{\partial v} \Delta = -2(2 - \tilde{\beta}^S(\phi^S))\tilde{\beta}^S(\phi^S) + \frac{2}{2v^2} \left(\frac{1}{2} + \rho v \left(\sigma_B^2 + \frac{\mu^2}{4}\eta\right)\right) \]
\[ \times 2(\beta^o_B(0, 0))^2 \left(1 - \frac{1}{2} + \rho v \left(\sigma_B^2 + \frac{\mu^2}{4}\eta\right)\right) - 2\tilde{\beta}^S(\phi^S) + (\tilde{\beta}^S(\phi^S))^2 \]
\[ = \beta^o_B(0, 0)[2 - \beta^o_B(0, 0)] - \tilde{\beta}^S(\phi^S)[2 - \tilde{\beta}^S(\phi^S)] \]
\[ > 0, \]
because $\beta(2 - \beta)$ is increasing in $\beta$ for any $\beta \in [0, 1]$ and $0 \leq \tilde{\beta}^S(\phi^S) < \beta^B_0(1, 1) < \beta^B_0(0, 0) < 1$. Further, $\lim_{v \to 0} d_2 = +\infty$ and, using L’Hopital rule, $\lim_{v \to 0} d_1 = -\frac{\phi}{2}(\tilde{\beta}^S(\phi^S))^2 \sigma^2_B - \frac{\phi^2}{8}(\tilde{\beta}^S(\phi^S))^2(\mu + 2)^2 < 0$. Hence $\lim_{v \to 0} D \to -\infty$ implying that there exist a threshold $\overline{v} > 0$, such that $D < 0$ if $v < \overline{v}$. To summarize, for given $\sigma^2_B$, there exist thresholds $\overline{\delta} \geq 0$ and $\overline{v} > 0$ such that, if $\sigma^2_A > \sigma^2_B + \overline{\delta}$ and $v < \overline{v}$, the principal prefers bundling.

**Proof of Proposition 5:** We first characterize the managers’ project effort choices under the split and bundling regimes:

**Lemma A 1** For scalable project efforts ($e \in [0, \overline{e}]^2$) and $i, \ell = A, B, j = S, \ell$:

(a) For given $\beta$, there exists a unique resulting project effort vector $e^S(\beta)$ under the split regime, and a unique effort vector $e^\ell(\beta)$ under bundling, such that $e^i(\beta)$ is monotonically non-decreasing in $\beta_i$.

(b) With endogenous contracts, $e^S_A < e^S_B < e^*_i$ under the split regime, and $e^\ell_i < e^*_i$ under bundling.

**Proof:** Part (a): Under the split regime, for given PPS, the best response for Manager $i$ to an anticipated effort $e_j$ by his counterpart, $e_i(e_j \mid \beta_i)$, is found by setting the first-order conditions equal to zero:

$$
\frac{\partial \Gamma(e \mid \beta)}{\partial e_i} - ge_i = \beta_i \left( \frac{q^*(\mu, e)}{2} - \frac{\mu q^*(\mu, e)}{4} \beta_i q^*(\mu, e) \right) - ge_i = 0
$$

$$
\iff e_i(e_j \mid \beta_i) = \frac{(e_j + \mu) \left( \frac{1}{2} - \frac{\mu}{4} \beta_i \right)}{\frac{g}{\beta_i} + \frac{\mu}{4} \beta_i - \frac{1}{2}}, \quad i = A, B, \; j \neq i.
$$

Because $e_i(e_j = 0 \mid \beta_i) > 0$, and $\frac{d}{de_j} e_i(e_j \mid \beta_i) < 1$ for any $e_j, \beta, i$ and $j \neq i$, we have a unique Nash equilibrium in investments for any PPS. (To bound the slope of the reaction curves by one, we use the fact that $g > 6$.) Solving for the equilibrium in closed form:

$$
e_i^S(\beta) = \frac{\beta_i \mu \left( \frac{1}{2} - \frac{\mu}{4} \beta_i \right)}{g + \frac{\mu}{4} \left( \beta_i^2 + \beta_j^2 \right) - \frac{3}{2}(\beta_i + \beta_j)}, \quad i = A, B, \; j \neq i.
The equilibrium efforts are increasing in either manager's PPS:
\[
\frac{\partial e^S_i(\beta)}{\partial \beta_i} = \frac{2\mu(1 - \rho \eta \beta_i)(4g + \rho \eta \beta^2_i - 2\beta_i)}{(\rho \eta (\beta^2_i + \beta^2_j) - 2(\beta_i + \beta_j - 2g))^2} \propto g + \frac{\rho \eta}{4} \beta^2_i - \frac{\beta_i}{2} \geq g - \frac{1}{2} > 0,
\]
\[
\frac{\partial e^S_i(\beta)}{\partial \beta_j} = \frac{2\beta_i \mu(2 - \beta_i \rho \eta)(1 - \beta_j \rho \eta)}{(\rho \eta (\beta^2_i + \beta^2_j) - 2(\beta_i + \beta_j - 2g))^2} \propto (2 - \beta_i \rho \eta)(1 - \beta_j \rho \eta) \geq (2 - 1)(1 - 1) = 0.
\]

Under bundling, for given PPS, Manager \(\ell\) chooses effort \(e_i\) by setting the first-order conditions equal to zero:
\[
\frac{\partial \Gamma(e | \beta \ell)}{\partial e_i} - ge_i = \frac{\beta \ell \mu}{2} \left( q^*(\mu, e) - \frac{\rho \eta}{4} \beta \ell q^*(\mu, e) \right) - ge_i = 0, \quad i = A, B.
\]
Solving for the equilibrium in closed form:
\[
e_i(\beta \ell) = \frac{\beta \ell \mu}{2} \left( \frac{1}{2} - \frac{\rho \eta}{4} \beta \ell \right), \quad i = A, B.
\]
The comparative statics follows immediately.

**Part (b):** Proving that \(e^S_A < e^S_B < e^*\), \(i = A, B\) and \(e^\ell_i < e^*_i\) proceeds by using arguments as in the proofs of Lemma 2b and Lemma 3b, respectively.

The remainder of the proof of Proposition 5 follows similar steps as that of Proposition 1 and is available upon request.

**Proof of Proposition 6:** We first characterize the optimal contracts and equilibrium project choice outcomes under the two delegation regimes in Lemmas A 2 and A 3. Then we compare the regime performance.

Under the split regime, Manager A, who faces greater general uncertainty and thus lower PPS, is the bottleneck in terms of eliciting project efforts. Hence, \(e^S(\beta) = (1, 1)\) constitutes an equilibrium for \(\gamma\) low enough such that
\[
\Gamma(1, 1 | \beta_A) - \gamma \geq \Gamma(1, 0 | \beta_A).
\]
Denote by \(\gamma_{11}(\beta_A)\) and by \(\tilde{\beta}^S(\gamma)\) the effort cost and PPS, respectively, at which (33) becomes binding, and by \(\gamma^S \equiv \gamma_{11}(\beta^o(1, 1))\) the effort cost level up to which \(e^B = (1, 1)\) obtains without any adjustments required to the benchmark PPS.

The optimal contract under the split regime is as follows.

48
Lemma A 2 (Split regime, project efforts) Consider lumpy project efforts $(e \in \{0,1\}^2)$, and suppose (20) holds. Under the split regime, there exists a unique threshold $\gamma^S \in (\gamma^S, \gamma^*)$, such that:

(a) If $\gamma \in (\gamma^S, \gamma^*)$, then $\beta_A = \tilde{\beta}^S(\gamma) > \beta_A^o(1,1)$ and $\beta_B = \max\{\beta_B^o(1,1), \tilde{\beta}^S(\gamma)\}$. Both managers exert project effort, as in the benchmark case.

(b) If $\gamma \in (\gamma^*, \gamma^*)$, then $\beta_i = \beta_i^o(0,0) > \beta_i^o(1,1)$, $i = A, B$. Neither manager exerts project effort, whereas $e^* = (1,1)$ in the benchmark case.

Proof of Lemma A 2: We begin by deriving a sufficient condition to rule out asymmetric project effort equilibria under the split regime. Recall that $e^S(\beta) = (1,1)$ is an equilibrium for $\gamma \leq \gamma_{11}(\beta)$, whereas $e^S(\beta) = (0,0)$ is an equilibrium for $\gamma > \gamma_{00}(\beta)$. If $\gamma_{11}(\beta) - \gamma_{00}(\beta) < 0$, then $e^S(\beta) = (1,0)$ can be a Nash equilibrium for any $\gamma \in (\gamma_{11}(\beta), \gamma_{00}(\beta))$. On the other hand, if $\gamma_{11}(\beta) - \gamma_{00}(\beta) \geq 0$, then for any $\gamma \in (\gamma_{00}(\beta), \gamma_{11}(\beta))$, both $(1,1)$ and $(0,0)$ can be equilibria simultaneously.

$$
\gamma_{11}(\beta) - \gamma_{00}(\beta) = \Gamma(1,1 \mid \beta) - \Gamma(1,0 \mid \beta) - (\Gamma(1,0 \mid \bar{\beta}) - \Gamma(0,0 \mid \bar{\beta}))
$$

$$
= \frac{\beta}{4} \left( E[(q^*(\theta, 1, 1))^2 - (q^*(\theta, 1, 0))^2] - \frac{\rho \eta}{2} (q^*(\mu, 1, 1))^2 - (q^*(\mu, 1, 0))^2 \right)
$$

$$
- \frac{\beta}{4} \left( E[(q^*(\theta, 1, 0))^2 - (q^*(\theta, 0, 0))^2] - \frac{\rho \eta}{2} (q^*(\mu, 1, 0))^2 - (q^*(\mu, 0, 0))^2 \right)
$$

$$
= \frac{1}{4} \left[ \beta(2\mu + 3) \left(1 - \frac{\beta \rho \eta}{2}\right) - \beta(2\mu + 1) \left(1 - \frac{\beta \rho \eta}{2}\right) \right].
$$

This term is proportional to:

$$
\frac{\beta(2\mu + 3)}{4} \left(1 - \frac{\beta \rho \eta}{2}\right) - \frac{\beta(2\mu + 1)}{4} \left(1 - \frac{\beta \rho \eta}{2}\right) > \frac{\beta(2\mu + 3)}{4} - \frac{\beta(2\mu + 3)}{4} + 2\beta \left(1 - \frac{\rho \eta}{2}\right),
$$

which is positive if

$$
\frac{\beta - \beta}{\beta} \leq \frac{2 - \rho \eta}{2\mu + 3}.
$$

(34)
Suppose (34) holds. For any $\gamma \in (\gamma_{00}(\beta), \gamma_{11}(\beta))$, to predict which of the symmetric equilibria, $(1,1)$ or $(0,0)$, the managers will play, by Theorem 7 in Milgrom and Roberts (1990), we can again ignore the shirking equilibrium. Now note that the left-hand side of (34) in the optimal solution under the split regime is bounded from above by $(\beta_{MH} - \beta_{MH}^*)/\beta_{MH}$; therefore a sufficient condition for asymmetric equilibria not to arise in equilibrium is Condition (20), which holds for sufficiently small uncertainty differential, $\sigma^2_A - \sigma^2_B$, and small $\eta$.

Assuming (20) holds, under the split regime, the principal will choose the optimization program with the greater value from among the following:

\[
\mathcal{P}_{11}^S \quad (\text{Induce effort under split regime}): \quad \text{For any } \gamma \in (\gamma^S, \gamma^*], \max_{\beta} W(\beta \mid e = (1,1)), \text{ subject to: (3) and } \beta_i \geq \tilde{\beta}^S(\gamma), \text{ for any } i.
\]

\[
\mathcal{P}_{00}^S \quad (\text{Forestall effort under split regime}): \quad \text{For any } \gamma \in (\gamma^S, \gamma^*], \max_{\beta} W(\beta \mid e = (0,0)), \text{ subject to: (3) and } \beta_i < \tilde{\beta}^S(\gamma), \text{ for any } i.
\]

The remainder of the proof follows similar steps as the proof of Proposition 2. The only difference is that $\gamma^S$ is defined as the minimum of (i) the fixed cost threshold above which the principal finds it not worthwhile inducing efforts, and (ii) the fixed cost threshold for which $\tilde{\beta}^S(\gamma) = 1$. (Note that beyond the latter fixed cost threshold the manager will no longer exert effort.)

Conditional on bundling, decision rights will be given to Manager B. Given $\beta_B$, Manager B will choose $(1,1)$ if and only if

\[
\Gamma(1,1 \mid \beta_B) - 2\gamma \geq \Gamma(0,0 \mid \beta_B), \tag{35}
\]

or equivalently, if $\gamma \leq \gamma_{11}^{t=\beta}(\beta_B) \equiv \frac{1}{2}[\Gamma(1,1 \mid \beta_B) - \Gamma(0,0 \mid \beta_B)]$; and $(0,0)$ otherwise. \footnote{Because we have assumed similar functional forms for all key constructs across the (monetary and personally costly) input scenarios, the effort cost threshold equals $\gamma_{11}^{t}(\beta_B) = \beta_0\phi_{11}(\beta_B)$, with $\phi_{11}(\beta_B)$ as defined by (11). Only the optimal assignment of decision rights will differ from that for monetary inputs. This reflects the fact that project effort costs are incurred privately by the manager. In the benchmark solution, on the other hand, the fixed cost threshold $\gamma^* = \phi^*$, as in Section 4.2, because ex ante the principal ultimately pays for all project input costs, whether monetary in nature or effort disutility.} Denote by $\tilde{\beta}^{t=\beta}(\gamma)$ the PPS that makes (35) binding for given $\gamma$, and by
\[ \gamma_{t,B} = \gamma_{t1}(\beta_{o}(1, 1)) \] the fixed cost level up to which bundling would implement high efforts absent any PPS adjustment, i.e., if \( \beta_B = \beta_{oB}(1, 1) \).

The optimal contract under bundling is described as follows.

**Lemma A 3 (Bundling, project efforts)** For lumpy project efforts (\( e \in \{0, 1\}^2 \)), under bundling Manager B (low general uncertainty) is designated investment center manager, and there exists a unique threshold \( \gamma_{t,B} \in (\gamma_{t,B}, \gamma^*) \) such that:

(a) If \( \gamma \in (\gamma_{t,B}, \gamma_{t,B}) \), then \( \beta_B = \tilde{\beta}_{t,B}(\gamma) > \beta_{oB}(1, 1) \) and \( \beta_A = \beta_{oA}(1, 1) \). Manager B chooses (1, 1), as in the benchmark case.

(b) If \( \gamma \in (\gamma_{t,B}, \gamma^*) \), then \( \beta_i = \beta_{o}(0, 0) > \beta_{o}(1, 1) \), \( i = A, B \). Manager B chooses (0, 0), whereas \( e^* = (1, 1) \) in the benchmark case.

**Proof of Lemma A 3**: Follows similar steps as that of Proposition 2 and is available upon request.

With these preliminaries in place, the proof of the performance comparison follows similar steps as that of Proposition 4 and is available upon request.
References


Friedman, H (2014). “Implications of power: When the CEO can pressure the CFO to bias reports.” *Journal of Accounting and Economics* 58, 117-141.


