Optimal Reporting When Additional Information Might Arrive*

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Abstract: We study how the potential for discretionary disclosure affects the way a firm designs its reporting system. In our model, the firm's primary but nonexclusive concern is to induce beliefs that exceed a threshold. Such thresholds arise in numerous contexts, including investing decisions, liquidation/continuation choices, covenants, audits, impairments, listing requirements, index inclusion, credit ratings, analyst recommendations, and stress tests. The optimal reporting system is characterized by informative good reports when the threshold is high and, potentially, uninformative reports when the threshold is low. Under an optimal impairment-type reporting system, the likelihood of reported impairments and the information content of non-impairment reports both increase in the probability of the firm observing private information. We provide a novel motivation for the quiet period around an IPO and empirical predictions relating the probability of discretionary disclosure to the properties of financial reports. In extensions, we consider disclosure mandates, report manipulation, endogenous thresholds, and alternative payoff functions.

Keywords: Bayesian persuasion, discretionary disclosure, mandatory disclosure, verifiable messages, commitment

1. Introduction

We explore interdependencies between the ex ante design of a reporting system and discretionary disclosure of subsequently received private information. While the design of public reporting systems and discretionary disclosure strategies have independently received considerable attention in the literature, few studies have examined interactions between these alternative means of conveying information to investors. These interactions are important because firms plausibly consider the entire disclosure environment when making financial reporting decisions. Broadly speaking, firms' discretionary disclosure strategies can depend on previously reported information, and the potential subsequent receipt and disclosure of private information can change the firm's incentives when designing a reporting system.

Our model consists of a sender-receiver game played between a firm and a representative investor. The firm's payoff depends on the investor's expectations of a state variable (e.g., profits from continuing operations, solvency, liquidity, enterprise value, the value of a specific piece of collateral, or operating cash flows). The primary but not exclusive concern of the firm is to induce beliefs that meet a predetermined threshold. Such thresholds arise in numerous contexts, including investing decisions, liquidation/continuation choices, listing requirements, index inclusion, credit ratings, debt covenants, analyst recommendations, bank stress tests, and audits.[1,2]

The firm can influence investors' beliefs through two channels. First, the firm commits ex ante to a design of a financial reporting system that provides reports about the realized state.[3] Second, after the reporting system is in place, the firm privately observes the state with some exogenous probability. Following such an observation, the firm can withhold its private information or disclose it, as in Dye (1985a) and Jung and Kwon (1988) (DJK hereafter). Reporting system design choices and report realizations may either accommodate

[1]While we refer to the receiver as an 'investor,' our model also applies to contexts in which the receiver is an information intermediary or regulator such as an analyst or bank examiner.

[2]Several prior analytical studies provide contexts that lead to threshold-based incentives for firms (e.g., Homstrom and Tirole 1997, Göx and Wagenhofer 2009), with the threshold increasing as agency problems get worse or the investor's outside opportunities improve. We explore these in Section 5.3.

[3]Directly corresponding to the context of our model, empirical findings support recourse to financial reporting practices as a means of boosting the likelihood of meeting thresholds upon which the value of the firm may depend (e.g., Bartov et al. 2001; Press and Weintrop 1990; Sweeney 1994; Dichev and Skinner 2002; Kim and Kross 1998; Ramanna and Watts 2012; Chen et al. 2015; Bonachi et al. 2015).

or dissuade subsequent disclosures of private information, and the prospect of such disclosures influences the ex ante design of the reporting system.

Our main results describe the firm's optimal reporting system and disclosure strategies following each possible report realization. In choosing its reporting system, the firm seeks to maximize the probability of meeting or exceeding the investor's threshold. The properties of the optimal reporting system depend on whether the investor's threshold is relatively low or high. When the threshold is low, the firm can set an uninformative reporting system. Following nondisclosure in the discretionary disclosure subgame, the investor's expectation of the state exceeds the low threshold. The firm is able to achieve its first-best payoff in that it avoids, with certainty, the cost of missing the threshold. When the threshold is high, and reports are uninformative, nondisclosure would cause beliefs about the firm to fall below the threshold. To reduce the probability of this happening, the firm sets an optimal reporting system that produces a good report for all states above an endogenous cutoff (i.e., pools high states). The key property of this reporting system is that the investor's threshold is met following a good report and nondisclosure. Following any other report realization, however, posterior beliefs about the firm fall below the investor's threshold. When the threshold is high, therefore, the firm bears the loss from missing the threshold with some positive probability. As an example, a system that reports impairments for low asset values and nothing for high values, with the impairment cutoff appropriately set, would be optimal in our setting.

We find that the endogenous cutoff separating states that produce the good report from all other states is increasing in the investor's threshold and the firm's probability of receiving information. Our result on the investor's threshold parallels Göx and Wagenhofer's (2009) result that increasing a lender's required collateral (i.e., a lender's threshold in a firm's reported asset value) causes the firm to increase its impairment cutoff. Our finding related to the firm's probability of receiving private information provides a novel implication for how a property of the discretionary disclosure environment can affect a firm's optimal reporting choices. As in DJK, an increase in the probability of being privately informed causes firms to be penalized more heavily for nondisclosure. This pushes the investor's belief following a good report and nondisclosure below the investor's threshold. To counteract this effect, the firm has to adjust the reporting system such that good reports imply higher states and in

this sense become more informative.

We also find that more favorable distributions of possible states are associated with lower cutoffs for good reports, implying less informative good reports that are produced more frequently. Furthermore, when the threshold is high, we find that the firm is strictly better off as its probability of being privately informed decreases. The firm would prefer, in fact, not to receive private information at all, as discretionary disclosures based on private information limit the firm's ability to influence investors via the reporting system. At an institutional level, the firm has incentives to support an enforced commitment to non-disclosure, as in a quiet period around an initial public offering.[4]

Focusing on an optimal impairment-type reporting system, in which low values are reported directly (e.g., an asset impaired down to net realizable value) and high values are combined (e.g., an asset left at historical cost), we find that the probability of discretionary disclosure is increasing in the probability that the firm observes private information and decreasing in the investor's threshold.[5] A higher probability of observing private information directly increases the probability with which the firm provides a discretionary disclosure. A higher threshold causes the firm to increase the good report cutoff. This change in the reporting cutoff shrinks the region for which the firm does not provide an impairment report, yielding fewer discretionary disclosures.

Our results involving an optimal impairment-type reporting system may help explain mixed empirical findings. Francis et al. (2008) and Lennox and Park (2006) find greater voluntary disclosure when earnings quality is high and when earnings are more informative, respectively. This is consistent with the effects of an increase in the probability that the firm observes private information, which increases the probability of discretionary disclosure and causes the firm to choose a reporting system that produces more informative good reports. Seemingly in contrast, Guay et al. (2016) find that discretionary disclosures are more common when financial statements are more complex, where complexity is taken as

[4]Our results also provide an alternative perspective on the effects of the regulatory intervention limiting voluntary disclosure. Abudy and Shust (2019), focusing on trading volume and liquidity, discuss an example in which the Israeli Securities Authority prohibited oil and gas firms from providing certain types of discretionary disclosures.

[5]Directional effects of changes in the investor threshold on the probability of discretionary disclosure depend on what type of optimal reporting system the firm implements. Analysis of special cases is available upon request.

a negative indicator of quality. This is consistent with the effects of lowering the investors' threshold, which increases the probability of discretionary disclosure and makes the good report both more likely and less informative.

We examine several extensions. First, we characterize an optimal reporting system under a mandatory disclosure regime in which the firm must disclose its private signal whenever it is obtained. We find that the firm prefers the discretionary disclosure regime. Second, we discuss the sensitivity of our results to alternative specifications of the firm's objective function. Third, we provide mechanisms for endogenizing the investor's threshold, allowing it to emerge from an agency issue or formulating it as a policy tool for getting firms or managers to be more forthcoming with private information, as a board of directors or stock exchange might wish to do. Fourth, we incorporate report manipulation, and illustrate how allowing the firm to manipulate tends to make the firm worse off in expectation.

2. Related literature

Several studies have examined the optimal design of a financial reporting system absent discretionary disclosures (e.g., Arya et al. 1997; Göx and Wagenhofer 2009; Bertomeu and Cheynel 2015). In the spirit of Bayesian persuasion models as applied in our analysis,[6] Göx and Wagenhofer (2009) characterize an equilibrium in which a pooling of high states above a threshold with some states below the threshold maximizes the probability of generating a "good" report that induces posterior beliefs above the threshold. They interpret this result, in the context of information about an asset used as loan collateral, as an "impairment-type" reporting system, thereby lending theoretical support to an accounting policy often observed in practice. Bertomeu and Cheynel (2015) extend Göx and Wagenhofer (2009) by allowing for the firm to sell the asset. They find that an "appreciation-type" reporting system, in which the firm reports high values and pools lower states, can obtain in equilibrium when the asset's resale value exceeds its value in use.

As well, a number of studies have examined interactions between financial reporting systems and voluntary disclosure. Einhorn (2005) considers a model in which a firm observes

[6]Bayesian persuasion models are a subset of sender-receiver games with an ex ante commitment to a reporting system popularized by Kamenica and Gentzkow (2011). Applications of Bayesian persuasion in recent accounting literature include Bertomeu et al. (2019), Huang (2016), and Michaeli (2017).

two potentially correlated signals. While the disclosure of one of the signals may be mandatory, the disclosure of the second signal is at the discretion of the firm. Her analysis takes the underlying properties of the signals as given and focuses on how properties of the former signal affect the firm's disclosure strategy. In our model, the firm designs its reporting system before potentially receiving private information that it can verifiably disclose. Versano (2019) compares mandatory and discretionary disclosure of manipulable soft information. He shows that investors may benefit from firms having discretion over whether to disclose soft information. Dye (1985b) assumes firms choose a reporting system conditional on private information that if not revealed by the reporting system may be voluntarily disclosed. In his analysis, requiring more informative reports may induce less informative voluntary disclosure. In our model, choosing a more informative reporting system may be optimal to compensate for the prospect of non-disclosure lowering posterior beliefs in order to improve the odds of meeting the threshold.

In a stylized model with two components to the firm's value, Friedman et al. (2018) consider a setting in which a regulator, rather than the firm, chooses the reporting system and the firm chooses the probability of receiving private information. In choosing the reporting system, the regulator, concerned with total informativeness of the combined stages, balances the benefit of more informative reports against the negative effects such reports may have on the firm's incentives to gather private information. In contrast, the probability of receiving information in our model is exogenous and the firm (as opposed to a regulator) chooses a reporting system and disclosure strategy to maximize its expected payoff. Cianciaruso and Sridhar (2018) consider how the confirmatory role of financial reports influences prior voluntary disclosures in a dynamic setting.

Bertomeu et al. (2019) feature a setting in which the regulator designs the properties of the information that may or may not be observed by the firm, with the firm potentially choosing to disclose or withhold the information. In contrast, our model features the firm first designing a reporting system that always provides reports and then potentially disclosing a private signal whose properties are independent of the reporting system. Stocken and Verrecchia (2004) present a model that allows for ex post manipulation of reports with an ex ante choice of financial reporting system precision and subsequent receipt of a private signal. In their model, however, the private signal cannot be disclosed, and only influences

the degree to which the financial reports are manipulated.

3. Economic setting

We model a sender-receiver persuasion game whereby a firm (it) communicates information about a state of nature to persuade a representative investor (she) to take an action that affects the firm's payoff.

The state of nature is given by $\omega \in \Omega \equiv [\underline{\omega}, \overline{\omega}]$, $\underline{\omega} < \overline{\omega}$, with $\omega \sim F(\omega)$. $F(\cdot)$ is the cumulative distribution function (CDF) and $f(\cdot) = F'(\cdot)$ is the probability distribution function (PDF).[7] Both $F(\cdot)$ and $f(\cdot)$ are continuously differentiable with $f(\omega) > 0$, $\forall \omega \in \Omega$. Because we focus on information provision that causes the revision of beliefs, we refer to $F(\cdot)$ and $f(\cdot)$ as prior distributions. The state of nature represents, for instance, profits from continuing operations, solvency, liquidity, enterprise value, the value of a specific piece of collateral, or operating cash flows.

The firm can provide information to the investor through two communication channels. The first channel is a reporting system designed by the firm, which generates a public report $r \in R$ about the state of nature. The design of the reporting system is represented by an observable choice of the conditional distributions of r for each potential realization of $\omega \in \Omega$. Formally, the reporting system is a function $\boldsymbol{\beta} : \Omega \times R \to [0, 1]$, which identifies the probability that each feasible report is produced for each state realization. Report $r_j \in R$, when produced by reporting system $\boldsymbol{\beta}$, causes the investor to update her belief about ω to $F_{r_j,\boldsymbol{\beta}}(\omega)$. Let β_i^j be the probability that report r_j is produced when the state is ω_i.[8] We will refer to the firm's choice of $\boldsymbol{\beta}$ as the design of the reporting system. While we give the firm full control over the reporting system, we acknowledge that our predictions are nonetheless potentially constrained by standards that limit measurement choices under GAAP. An important feature of choices allowed by standards defining GAAP is the inclusion of those choices in publicly available financial reports, thereby satisfying the assumption that reporting system designs are observable in our model.

The second communication channel is discretionary disclosure. With probability $p \in$

[7] Our analysis also applies to distributions with unbounded support, i.e., allowing $\underline{\omega} \to -\infty$ and $\overline{\omega} \to \infty$. We only require that $E[\omega]$ exists and $F(\omega)$ is continuous.

[8] The subscripts $i, j \in \mathbb{R}$ are not meant to imply discreteness in Ω or R.

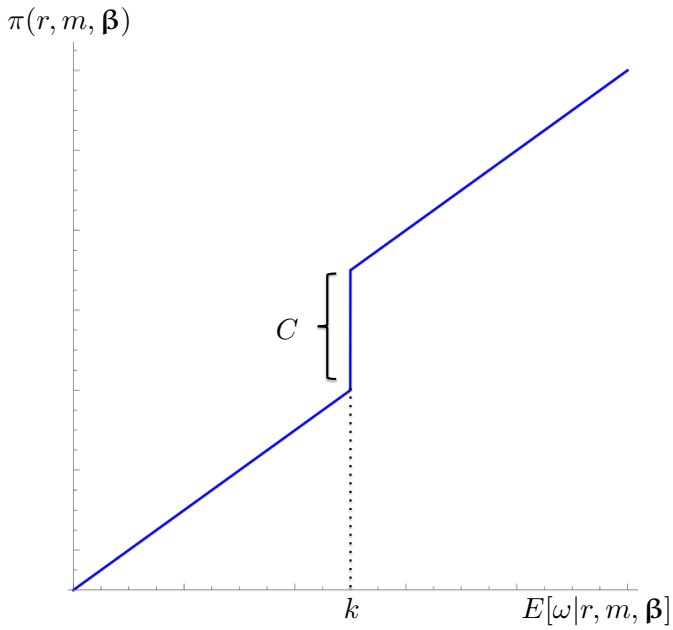

Figure 1: Firm's payoff

$(0, 1)$ the firm privately observes a signal, $s = \omega$, that perfectly reveals the state of nature, and with probability $1 - p$ it does not observe the signal. Upon observing the private signal, the firm can either send a truthful message, $m = s$, or withhold its information, i.e., $m = \varnothing$. If the firm does not observe a signal it has no choice but to send $m = \varnothing$. The firm cannot credibly communicate that it did not observe a signal. Importantly, the investor cannot determine, conditional on $m = \varnothing$, whether the firm did or did not observe a private signal.

After observing the report realization, r, and the firm's message, m, the investor forms a posterior expectation about the state, $E[\omega|r, m, \boldsymbol{\beta}]$, and takes an action that affects the firm. Formally, the firm's payoff is given by $\pi(r, m, \boldsymbol{\beta}) = E[\omega|r, m, \boldsymbol{\beta}] - C\mathbb{1}_{E[\omega|r,m,\boldsymbol{\beta}]<k}$, which consists of a portion linear in the investor's conditional expectation of the state, $E[\omega|r, m, \boldsymbol{\beta}]$, and a portion that reflects a discrete loss of $C > 0$ incurred if the investor expects that the firm's state is below a predetermined threshold $k \in \Omega$ (Figure 1). For most of the paper we assume that k is exogenously given. We examine various ways of endogenizing this threshold in Section 5.3.

The assumed stylized payoff function is a parsimonious yet flexible representation of multiple potential settings. For example, in the investment application described further in Section 5.3.1, a lender decides whether to invest in a project after forming a posterior

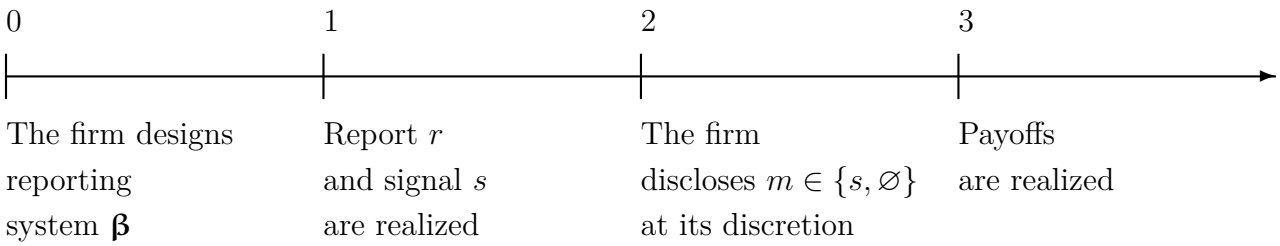

Figure 2: Timeline

about the firm's pledgable assets. Investment occurs if and only if the lender believes the pledgable assets are above a certain threshold, which can represent the lender's exogenous outside investment opportunity (Holmström and Tirole, 1997; Göx and Wagenhofer, 2009; Bertomeu and Cheynel, 2015). As a result, the firm's objective function is increasing in posterior beliefs and faces a discrete jump around a threshold.[9] We elaborate further on the motivation for the firm's objective and discuss alternative payoff functions in Section 5.2.

The timeline is shown in Figure 2. At date 0, the firm chooses the properties of the reporting system, $\boldsymbol{\beta}$. Investor beliefs at date 0 are described by the prior, $F(\omega)$. At date 1, the report, r_j, and the signal, s, are realized. Investor beliefs are updated to the intermediate distribution conditional on the reporting system and report, $F_{r_j,\boldsymbol{\beta}}(\omega)$. At date 2, an informed firm decides whether to truthfully disclose its private information, i.e., decides between sending $m = s$ or $m = \varnothing$. An uninformed firm sends $m = \varnothing$. Investor beliefs are updated again based on the message, which yields the posterior distribution, $F_{r_j,m,\boldsymbol{\beta}}(\omega)$. Finally, at date 3, the investor forms posterior beliefs upon which the firm's payoff depends.

An equilibrium in our model consists of the following: (i) a disclosure strategy that maximizes the firm's expected payoff, conditional on the investor's conjecture about the

[9]Other applications (delisting, index exclusion, credit rating downgrade and bank stress tests) also exhibit features consistent with the assumed payoff function. A firm is delisted and incurs significant cost if the market price (representing investors' posteriors about firm value) falls below a predetermined threshold (1$ at NYSE and NASDAQ). A firm may face removal from an index such as the Russell 1000 if it fails to meet the selection criteria which we take to be associated with low posterior beliefs related to price. A credit downgrade from investment to speculative-grade may also follow from sufficiently low beliefs. Macey et al. (2008), Klas et al. (2016), Beneish and Gardner (1995), and Kisgen (2006) provide evidence, respectively, of delisting, index exclusion, and credit rating downgrades being costly for firms. In the bank stress test application, banks face prospects ranging from depositor runs to FDIC receivership if depositors', creditors', or regulators' beliefs about the bank's future prospects are sufficiently low.

firm's disclosure strategy, following any possible report; (ii) a conjectured disclosure strategy by the investor consistent with the firm's disclosure strategy following any possible report; and (iii) a reporting system that, conditional on (i) and (ii), maximizes the firm's expected payoff. Parts (i) and (ii) define a rational expectations equilibrium in the disclosure subgame, as in DJK.

Throughout the paper we refer to two benchmarks. The ***disclosure benchmark*** captures a setting with only the disclosure stage, as in DJK. Such a setting can be approximated by allowing the reporting system to become uninformative, i.e., there is one report, r_j, with $\beta_i^j = 1,\ \forall i$. The ***reporting benchmark*** captures a setting with only the reporting stage, as in Kamenica and Gentzkow (2011) and Göx and Wagenhofer (2009). Such a setting can be approximated by disallowing any receipt of private information, i.e., setting $p = 0$.

4. Main analysis and results

We solve the model by backward induction. We first derive the optimal disclosure policy conditional on a report realization and the reporting system chosen by the firm. Next, we consider the firm's reporting system design, taking into account its subsequent optimal disclosure strategy.

4.1. Disclosure strategy for a given report

A firm that observes no signal has no choice but to send a null message, $m = \varnothing$. A firm that observes an informative signal, s, decides whether to truthfully disclose it at date 2. In considering the firm's choice, note that any given report realization, r_j, and reporting system properties, $\boldsymbol{\beta}$, create a conditional cumulative distribution function $F_{r_j,\boldsymbol{\beta}}(\omega)$, which we refer to as the intermediate belief, i.e., the belief about ω upon observing r_j for a given $\boldsymbol{\beta}$, but before observing m. If the firm discloses the signal that it observed, the investor's posterior expectation equals the observed signal, i.e., $E[\omega|r, m = s, \boldsymbol{\beta}] = s$. If the firm withholds the observed signal, the investor's posterior expectation, $E[\omega|r, m = \varnothing, \boldsymbol{\beta}]$, incorporates the probabilities that the firm had no private information and that it observed a signal and chose not to disclose it. When deciding whether to disclose the observed signal realization s, the firm compares the payoff it receives when it discloses the signal, $\pi(r, m = s, \boldsymbol{\beta})$, with the

payoff it receives when it withholds the signal, $\pi(r, m = \varnothing, \boldsymbol{\beta})$, and chooses the message that yields a higher payoff.

Lemma 1 *For a given report realization r_j and reporting system design $\boldsymbol{\beta}$, there exists a unique disclosure strategy defined by a cutoff, $t_{r_j,\boldsymbol{\beta}} \in \Omega$, such that the firm discloses all signals $s \geq t_{r_j,\boldsymbol{\beta}}$ and withholds all others.*

All proofs are in the appendix. Lemma 1 follows from DJK. If a report r_j induces a differentiable distribution $F_{r_j,\boldsymbol{\beta}}(\omega)$ on $\Omega_j = \left[\underline{\omega}_j, \overline{\omega}_j\right]$, then the disclosure cutoff $t_{r_j,\boldsymbol{\beta}}$ is defined by the following restatement of equation (7) from Jung and Kwon (1988):

$$E[\omega | r_j, \boldsymbol{\beta}] = t_{r_j,\boldsymbol{\beta}} + \frac{p}{1-p} \int_{\underline{\omega}_j}^{t_{r_j,\boldsymbol{\beta}}} F_{r_j,\boldsymbol{\beta}}(\omega)\, d\omega. \tag{1}$$

If r_j induces a non-differentiable distribution the solution is similar but with "=" in equation (1) replaced by "$\leq$". As we show later, the set of optimal reporting systems always includes reporting systems that induce only conditional distributions with connected support, which are either perfectly informative or differentiable.

Following any report realization, r_j, as is standard in DJK-style voluntary disclosure models, the disclosure cutoff $t_{r_j,\boldsymbol{\beta}}$ is decreasing in p and independent of k. A higher probability that the firm was endowed with information results in a lower posterior expectation upon non-disclosure. Hence, the firm discloses more observed states, which increases the probability of disclosure. The independence of the disclosure cutoff with respect to k reflects the fact that, as long as the firm's payoff is an increasing function of the investor's conditional expectation, nonlinearities in the firm's payoff do not affect its optimal disclosure policy.

4.2. Optimal reporting system

At date 0, the firm chooses the design of the reporting system, $\boldsymbol{\beta}$, anticipating the receipt and discretionary disclosure of private information. The firm's expected payoff at date 0 is: $E[\pi] = E[E[\omega|r, m, \boldsymbol{\beta}] - C\mathbb{1}_{E[\omega|r,m,\boldsymbol{\beta}]<k}] = E[\omega] - C(1 - \Pr(E[\omega|r, m, \boldsymbol{\beta}] \geq k))$. Its objective is therefore to maximize the probability that $E[\omega|r, m, \boldsymbol{\beta}] \geq k$, subject to Bayesian plausibility

and optimal discretionary disclosure:

$$\max_{\boldsymbol{\beta}} \quad \Pr(E[\omega|r, m, \boldsymbol{\beta}] > k) \tag{2}$$

$$\text{s.t.} \quad E[F_{r_j,\boldsymbol{\beta}}(\omega|r, \boldsymbol{\beta})] = F(\omega), \ \forall \omega \in \Omega, \text{ and} \tag{3}$$

$$\text{Disclose } s \text{ following Lemma 1.} \tag{4}$$

The Bayesian plausibility constraint in (3) ensures that the expectation of the report-conditional distributions equals the prior distribution, which must hold if the investor updates following Bayes' rule.

It is useful to introduce the following definition, which we use to facilitate the presentation of our main results regarding the optimal reporting system:

Definition 1 *The investor's threshold is* ***high*** *if $k > \widehat{k}$, where $\widehat{k} \in (\underline{\omega}, \overline{\omega})$ is uniquely defined by the condition on primitives $\int_{\underline{\omega}}^{\overline{\omega}} \omega f(\omega) d\omega - \left(\widehat{k} + \frac{p}{1-p} \int_{\underline{\omega}}^{\widehat{k}} F(\omega) d\omega\right) = 0$. Otherwise, if $k \leq \widehat{k}$, the investor's threshold is* ***low****.*

The condition defining $\widehat{k}$ implies that, with an uninformative reporting system (alternatively, in the disclosure-only benchmark), the firm would prefer to disclose all private signals greater than $\widehat{k}$ and withhold all signals less than $\widehat{k}$. Note that an increase in the probability of observing a private signal, p, causes $\widehat{k}$ to decrease because a higher p causes investors' beliefs to react more negatively to non-disclosure. This makes it more likely that the threshold is above the non-disclosure belief, i.e., that $k > \widehat{k}$. Definition 1 is important for determining the types of optimal reporting systems, as described by the following Proposition.

Proposition 1

(i) When k is low, any optimal reporting system will have $E[\omega|r = r_j, m = \varnothing] \geq k$ for every report produced with positive probability, i.e., for every r_j with $\beta_i^j > 0$ for some $\omega_i \in \Omega$. An uninformative reporting system satisfies this criterion. The investor's threshold is always met and the firm's expected payoff is $E[\omega]$.

(ii) Otherwise, when k is high, every optimal reporting system is characterized by a unique interior reporting cutoff $\widehat{\omega} \in (\underline{\omega}, k)$, such that the reporting system generates a good

report r_g with probability 1 for all $\omega_i \geq \widehat{\omega}$. In equilibrium, the disclosure cutoff after a good report realization equals the investor's threshold, i.e., $t_{r_g,\boldsymbol{\beta}} = k$, so

$$\int_{\widehat{\omega}}^{\overline{\omega}} \frac{\omega f(\omega)}{1 - F(\widehat{\omega})\, d\omega} = k + \frac{p}{1-p} \int_{\widehat{\omega}}^{k} \frac{F(\omega) - F(\widehat{\omega})}{1 - F(\widehat{\omega})} d\omega. \tag{5}$$

The investor's threshold is always met following r_g and never met following any other report $r_j \neq r_g$. The firm's expected payoff is $E[\omega] - C \times F(\widehat{\omega})$.

Proposition 1 describes the optimal reporting systems that could be chosen by the firm in equilibrium. When the investor's threshold, k, is low, it is met with an uninformative reporting system. In such a case, the condition $k \leq \widehat{k}$ ensures that the investor's expectation of ω conditional on non-disclosure will be above the threshold, i.e., $E[\omega|m = \varnothing] \geq k$. The firm will optimally disclose signals higher than the investor's threshold and avoid the cost, C, with both disclosure and non-disclosure. The firm thus achieves its first-best payoff of $E[\omega]$.[10]

If the investor's threshold, k, is high, an uninformative reporting system yields an optimal disclosure cutoff below k. The firm will miss the investor's threshold and face a cost C following any non-disclosure and following any disclosure of signals lower than k. To avoid this, the firm sets a reporting system that produces a good report for underlying states at or above $\widehat{\omega}$. The optimal disclosure policy following the good report is then to disclose all signals above k. This yields a posterior expectation of the state, conditional on the good report and nondisclosure, of precisely k. All other reports, $r_j \neq r_g$ produce posterior beliefs strictly below k, regardless of subsequent discretionary disclosure. The firm therefore avoids the discrete cost, C, following the good report, but incurs the cost whenever any other report is transmitted.[11]

[10]Note that an uninformative reporting system is not a unique optimum. The firm can also avoid inducing posterior expectations below the threshold by setting an informative reporting system as long as every report, when followed by nondisclosure, maintains a posterior expectation greater than the investor's threshold. An interesting example of such a reporting system is an appreciation-type reporting system that perfectly reports high states and pools low states into a bad report that retains posterior beliefs above the threshold following nondisclosure.

[11]Note that Proposition 1 implies that, with high k, any optimal reporting system will pool high states into a good report. By pooling together high signals and setting the disclosure cutoff at k, the firm ensures that the threshold is met with any message that follows r_g. For any $s \geq k$ the firm will disclose, and so $E[\omega|r = r_g, m = s, \boldsymbol{\beta}] = s > k$. For any $s < k$ the firm will withhold and so $E[\omega|r = r_g, m = \varnothing, \boldsymbol{\beta}] = k$. The treatment of states below $\widehat{\omega}$ is not crucial to the maximization of the firm's expected payoff.

Corollary 1 *When k is high, the reporting cutoff $\widehat{\omega}$ is increasing in k and p. An increase in $\widehat{\omega}$ lowers the probability of a good report but makes it more informative.*

An increase in the investor's threshold, k, requires fewer and higher state realizations to be pooled together in order for the threshold to be met. When k is high, this is achieved by setting a higher reporting cutoff, $\widehat{\omega}$, which is similar to the result in Göx and Wagenhofer (2009) that the firm's impairment threshold is increasing in the asset value required by a lender to make a loan to the firm (discussed further in Section 5.3.1).

Increasing the probability that the firm observes a private signal lowers the investor's expectation of the state in the event of non-disclosure. Then, it is more likely that the firm observed an unfavorable state and chose not to disclose it when p is higher. To ensure that the threshold is met with non-disclosure, the firm reports only states above a higher cutoff point as good. A higher probability of the firm observing private information thus causes the reporting cutoff to increase.

An increase in the reporting cutoff affects the information provided by good reports. Consider a firm facing high k that adopts an optimal impairment-type reporting system, in which r_g is interpreted as a non-impairment event and any report $r_j \neq r_g$ reflects an impairment. Alternatively, consider a firm-run stress test in which the firm either reports a passing grade of r_g or a failing grade of r_b, i.e., a binary reporting system. With such reporting systems, an increase in the reporting cutoff would imply a higher prior likelihood of a below-threshold event, i.e., an impairment report of $r_j = \omega < \widehat{\omega}$ or a stress test failure report of r_b. Furthermore, although the good report is produced less frequently, it can be viewed as better news, as it is produced by fewer and higher underlying states when the reporting cutoff increases. Empirically, we might thus expect to observe more positive reactions to good reports (i.e., non-impairment events or the passing of stress tests) for firms that are more likely to be privately informed.

The linear component in the firm's payoff implies differences in equilibrium disclosure strategies after report $r_i \neq r_g$ between any two possible optimal reporting systems. For example, under an impairment system all state realizations below the equilibrium reporting cutoff are reported, implying indifference over subsequent disclosure. With binary reporting, the linear component in the firm's payoff provides an incentive for disclosure by privately informed firms following the imperfectly-informative bad report, $r_b \neq r_g$. We discuss these

in Sections 4.3 and 4.4.

As shown in Proposition 1, part (ii), the firm's expected payoff at any optimal reporting system with high k is $E[\omega] - C \times F(\widehat{\omega})$. Applying Corollary 1 yields the following:

Corollary 2 *If k is high, the firm's expected payoff is decreasing in k and p.*

When the investor's threshold, k, increases, the firm is worse off because the probability of inducing a posterior expectation above the threshold decreases. An increase in the probability of being privately informed, p, makes the firm worse off because of the effect identified in Corollary 1. Specifically, when p increases, the belief following nondisclosure worsens. To counteract this and maintain posterior beliefs greater than the threshold following nondisclosure, the firm shifts $\widehat{\omega}$ upward, which improves the expectation conditional on the good report, both overall and jointly with a nondisclosure event. Shifting $\widehat{\omega}$ up, however, means that good reports are produced less frequently, lowering the firm's expected payoff.

Corollary 1 implies that, when the investor's threshold is high, the firm prefers never to be privately informed.[12] This preference is based on belief revision following nondisclosure. If the firm was prevented from disclosing any private information, there would be no belief revision following nondisclosure, which would have a similar effect as setting $p = 0$. Such a prohibition arises in the quiet period around an initial public offering and in some regulatory interventions (e.g., Abudy and Shust 2019). Corollary 1 suggests that firms with high k would support these prohibitions in order to avoid discretionary disclosures that could harm the IPO process. The preference for avoiding discretionary disclosure provides an alternative to the usual motivation for the IPO quiet period, which is to limit excessive promotion.

We next consider how changes in the prior distribution of ω affect the reporting cutoff $\widehat{\omega}$. We focus on the case when k is high because this ensures that the reporting system is informative. Changes in the distribution of ω when k is and remains low may have no effect if the reporting system is and remains uninformative, as in Proposition 1, part (i).

We focus on how shifts in stochastic dominance affect the choice of cutoff, $\widehat{\omega}$. Recall that $F_{r_g,\boldsymbol{\beta}}(\omega)$ is the distribution of ω conditional on report r_g and reporting system $\boldsymbol{\beta}$. For high k, $F_{r_g,\boldsymbol{\beta}}(\omega)$ is the right tail of $F(\omega)$, with support $[\widehat{\omega}, \overline{\omega}]$. Proposition 3 from Jung

[12] Absent discretionary disclosure, the firm can ex ante set the reporting system to induce the same distribution of beliefs as would be induced by any combination of reporting system and subsequent discretionary disclosure. However, the converse is not true, as the discretionary disclosure stage strictly constrains the set of beliefs that can be induced.

and Kwon (1988) implies that a first-order stochastic dominance (FOSD) or second-order stochastic dominance (SOSD) shift in $F_{r_g,\boldsymbol{\beta}}(\omega)$ will cause the disclosure cutoff $t_{r_g,\boldsymbol{\beta}}$ to shift up, allowing the firm to shift $\widehat{\omega}$ down while maintaining the condition, $t_{r_g,\boldsymbol{\beta}} = k$, that allows the threshold to be met following nondisclosure. However, FOSD and SOSD shifts in the prior distribution of states, $F(\omega)$, do not imply FOSD or SOSD shifts in the conditional distribution, $F_{r_g,\boldsymbol{\beta}}(\omega)$.[13]

To provide a general result on how properties of the prior distribution can affect the reporting cutoff, $\widehat{\omega}$, we use hazard ratio dominance (HRD), which is a stronger ordering on distributions than FOSD.[14]

Definition 2 *Let $F_1(\omega)$ and $F_2(\omega)$ be two distributions on Ω with PDFs $f_1(\omega)$ and $f_2(\omega)$, respectively. $F_1(\omega)$ hazard ratio dominates $F_2(\omega)$ if*

$$\frac{f_1(\omega)}{1 - F_1(\omega)} \leq \frac{f_2(\omega)}{1 - F_2(\omega)}, \quad \forall \omega \in \Omega. \tag{6}$$

The key property of hazard ratio dominance is that HRD implies FOSD on any upper tail. So, if $F_1(\omega)$ hazard ratio dominates $F_2(\omega)$, and x is some value of ω in the support of F_1 and F_2, then $F_1(\omega|\omega > x)$ first order stochastic dominates $F_2(\omega|\omega > x)$. This property, combined with Jung and Kwon's (1988) Corollary 3 result, yields the following.

Corollary 3 *Let $F_1(\omega)$ and $F_2(\omega)$ be two distributions on the same support Ω, with reporting cutoffs $\widehat{\omega}_1$ and $\widehat{\omega}_2$ and with high k under both distributions. If $F_1(\omega)$ hazard ratio dominates $F_2(\omega)$, then $\widehat{\omega}_1 \leq \widehat{\omega}_2$.*

Our result implies that more favorable prior distributions, in the sense of hazard ratio dominance, lead to reporting systems that allow more states to generate good reports. The intuition behind this corollary is that hazard ratio dominance implies that the beliefs (i.e., distribution of ω) following a good report are more positive. All else equal, this would shift

[13] Counterexamples are easy to construct, as FOSD and SOSD shifts in the initial distribution, $F(\omega)$ can be constructed in such a way as to yield arbitrary changes in the right tail, $F(\omega|\omega > \widehat{\omega})$. When the prior is uniformly distributed, SOSD shifts lead to monotonic effects on $\widehat{\omega}$ and the firm's expected payoff, but SOSD shifts when the prior is normally distributed can lead to non-monotonic effects on $\widehat{\omega}$ and the firm's expected payoff. Proof available from the authors.

[14] Readers may be more familiar with the monotone likelihood ratio property (MLRP), $\frac{f_1(\omega_a)}{f_2(\omega_a)} \geq \frac{f_1(\omega_b)}{f_2(\omega_b)}$, $\forall \omega_a \geq \omega_b$. MLRP is a stronger ordering on distributions than HRD, and MLRP implies HRD (Shaked and Shanthikumar 2007). We use HRD rather than MLRP because HRD is a weaker and more general condition.

the disclosure cutoff up. The firm optimally lowers the good report cutoff, $\widehat{\omega}$, to bring the disclosure cutoff, $t_{r_g,\boldsymbol{\beta}}$, back down to k, which increases the prior probability of avoiding the k-related cost.

4.3. Optimal impairment-type reporting system

Results related to the probability of discretionary disclosure depend on the properties of the optimal reporting system chosen by the firm. Proposition 1 provides conditions for an optimal reporting system, but these conditions may be satisfied by various reporting systems (e.g., both of the previously discussed impairment-type and stress-test/binary reporting systems may be optimal for the same set of parameters). In this section, we focus on results for an impairment-type reporting system when k is high. Recall that an optimal impairment-type reporting system is one in which all states larger than the reporting cutoff $\widehat{\omega}$ are mapped into a good, non-impairment report, r_g, and all states below the reporting cutoff result in an impairment report, such that $r_i = \omega_i$ for any $\omega_i < \widehat{\omega}$.

Impairment-type reporting systems are ubiquitous under U.S. GAAP and IFRS, and have received significant attention in prior literature. Göx and Wagenhofer (2009) focus on an impairment-type system as one that either reports the underlying measured state (an impairment) or provides no new information (a non-impairment event). This is also the most informative optimal reporting system when k is high, where more informative reporting systems report the underlying state more frequently.[15]

With an impairment-type reporting system, there is no discretionary disclosure following impairment reports of $r_j \neq r_g$.[16] The probability of discretionary disclosure is given by

$$\begin{aligned}
\Pr(\text{discretionary disclosure}) &= \Pr(r_g)\Pr(\text{discretionary disclosure}|r_g) \\
&= (1 - F(\widehat{\omega}))\, p \left(\frac{1 - F(k)}{1 - F(\widehat{\omega})}\right) \\
&= p\,(1 - F(k))
\end{aligned}$$

and depends directly on the exogenous parameters p and k and the prior cumulative distri-

[15] We use 'most informative' somewhat loosely here. Our notion of informativeness corresponds to providing the finest partition over the largest possible set of underlying states.

[16] We assume non-disclosure following a report that perfectly reveals the state. Disclosure after the investor knows the state ω, if provided, would be irrelevant to the investor.

bution function $F(\cdot)$. The result below is immediate.

Corollary 4 *When k is high, the probability of discretionary disclosure with an optimal impairment-type reporting system is increasing in p and decreasing in k.*

With an impairment-type reporting system, the probability of disclosure increases in p because of the increase in the probability of the firm being privately informed. Although the reporting cutoff, $\widehat{\omega}$ shifts, the decrease in the probability of a good report is offset by the increase in the probability of disclosure following the good report. An increase in k affects the probability of disclosure indirectly, through its effects on the reporting system. Specifically, with an exogenous reporting system, a shift in k would have no effect (see Section 4.1). However, with an optimal reporting system as described in Proposition 1, the firm sets the reporting system such that, if informed, it has incentives to disclose all signals above k. It is therefore the shift in the reporting system, driven by the influence of the discretionary disclosure subgame, that allows the investor's threshold to affect the probability of disclosure of private signals.

With an optimal impairment-type reporting system, the investor will know the state perfectly if $\omega < \widehat{\omega}$ (because an impairment-type reporting system reports all $\omega < \widehat{\omega}$ perfectly) and, with probability p, if $\omega > t_{r_g,\boldsymbol{\beta}} = k$ (because of discretionary disclosure). When p increases, the probability of the investor kowing the state with certainty increases for two reasons. First, the firm is more likely to observe the state and disclose it. Second, by Corollary 1, when p increases, $\widehat{\omega}$ shifts upward to allow a more positive good report to accommodate the more negative beliefs following non-disclosure. The region in which the firm provides a perfectly informative report (i.e., an impairment) thus increases. Interestingly, an increase in p, via this channel, yields a more informative reporting system because a higher $\widehat{\omega}$ implies that the reports generate a finer partition of states.

When k increases, there are offsetting effects. First, similar to the case with an increase in p, the positive effect of the increase in k on $\widehat{\omega}$ causes the firm to expand the impairment region. However, an increase in k causes the firm to adjust the reporting system in such a way that the optimal disclosure policy following a good report continues to be "disclose if $s \geq k$." Higher k thus implies less disclosure of private signals, all else equal. It turns out that this latter countervailing effect dominates the former so that the probability of the investor knowing the state with certainty is, in fact, decreasing with k.

4.4. Optimal binary reporting system

Binary reporting systems appear in several institutional contexts, including revenue recognition choices and pass/fail stress tests. They are also the least informative type of optimal reporting system when k is high. Applying Proposition 1, in an optimal binary reporting system with high k, all states larger than the reporting cutoff $\widehat{\omega}$ are mapped into a good report r_g and all states below $\widehat{\omega}$ are mapped into a bad report r_b. Because the bad report does not reveal the underlying state, the firm will play a discretionary disclosure game on the conditional distribution $F_{r_b,\boldsymbol{\beta}} = \frac{F(\omega)-F(\underline{\omega})}{F(\widehat{\omega})-F(\underline{\omega})} = \frac{F(\omega)}{F(\widehat{\omega})}$ following the bad report. Let $t_{r_b,\boldsymbol{\beta}}$ be the disclosure cutoff following a bad report as defined by equation (1). Overall, the firm discloses high and intermediate signals, i.e., discloses $s \in [t_{r_g,\boldsymbol{\beta}}, \overline{\omega}]$ following a good report and $s \in [t_{r_b,\boldsymbol{\beta}}, \widehat{\omega}]$ following a bad report, with $t_{r_b,\boldsymbol{\beta}} < \widehat{\omega} < t_{r_g,\boldsymbol{\beta}}$. The total probability of disclosure is

$$\begin{aligned} \Pr(\text{disclosure}) &= \Pr(r_g)\Pr(\text{disclosure}|r_g) + \Pr(r_b)\Pr(\text{disclosure}|r_b) \\ &= p\,(1-F(k)) + p\,(F(\widehat{\omega}) - F(t_{r_b,\boldsymbol{\beta}}))\,. \end{aligned} \tag{7}$$

The following corollary provides comparative statics for the probability of disclosure with an optimal binary reporting system when the threshold, k, is high.[17]

Corollary 5 *When k is high, the probability of disclosure with an optimal binary reporting system is increasing in p and can be non-monotonic in k.*

The fact that the probability of discretionary disclosure is increasing in the firm's likelihood of having private information comes as no surprise, but it is worth noting that there are countervailing effects. In particular, as p increases, the expectation following non-disclosure decreases. So, an increase in p causes the firm to increase $\widehat{\omega}$, which in turn causes the disclosure cutoff following a bad report, $t_{r_b,\boldsymbol{\beta}}$, to increase. This increase in $t_{r_b,\boldsymbol{\beta}}$ can reduce the probability of disclosure, as states $\omega \in [\underline{\omega}, t_{r_b,\boldsymbol{\beta}}]$ lead to non-disclosure following a bad report.

The reason for the potential non-monotonicity in k is that there are three offsetting effects. First, an increase in k causes the firm to shift $t_{r_b,\boldsymbol{\beta}}$ so as to maintain the disclosure cutoff equal to k following a good report, r_g. This decreases the probability of a good report

[17] Note that a binary reporting system can also be optimal if k is low, as Proposition 1, part (i) only requires that any report followed by non-disclosure maintains a posterior expectation above k.

and disclosure, equal to $p(1 - F(k))$. The increase in $\widehat{\omega}$ makes a bad report more likely, and amounts to a first-order stochastic dominance (FOSD) shift in the distribution implied by a bad report. The second effect of an increase in k is thus that the probability of being above the disclosure cutoff following a bad report, and the probability of producing a bad report, both increase. However, there is an offsetting third effect, which is that the disclosure cutoff following the bad report, $t_{r_b,\boldsymbol{\beta}}$, shifts up due to the increase in $t_{r_b,\boldsymbol{\beta}}$ (i.e., the FOSD shift in $F_{r_b,\boldsymbol{\beta}}(\omega)$). This increase in $t_{r_b,\boldsymbol{\beta}}$ tends to decrease the probability of disclosure. Overall, either the first and third (negative) effects or the second (positive) effect may dominate, depending on the shape of the prior distribution.

Figure 3 shows the probability of disclosure first increasing then decreasing in k for $\omega \sim \text{Beta}\,(1, 2)$ and $p = 0.4$, given an optimally-set binary reporting system. It provides a numerical example for the nonmonotonicity described in Corollary 5. With a binary reporting system, the probability of residual uncertainty about the state is simply one minus the probability of disclosure. It is therefore decreasing in p and can be non-monotonic in k.

5. Extensions

In this section we explore several variants of our main model. First, we examine a mandatory disclosure setting in which the firm is required to disclose its signal whenever received. We then consider alternative objective functions for the firm. Third, we allow for two ways of endogenizing the investors' threshold, allowing it to arise from an agency issue or an information-seeking regulator. Lastly, we consider the consequences of allowing the firm to manipulate reports.

5.1. Mandatory disclosure of private signals

We denote a mandatory disclosure regime as one that requires the firm to disclose its private signal, s, whenever it is observed. Before describing the mandatory disclosure setting, we first revisit the reporting benchmark without discretionary disclosure, i.e., as $p \to 0$. This setting is similar in spirit to Göx and Wagenhofer (2009). Without discretionary disclosure in the second stage, the firm would set $\widehat{\omega} = k$ whenever $E[\omega] < k$ and provide uninformative

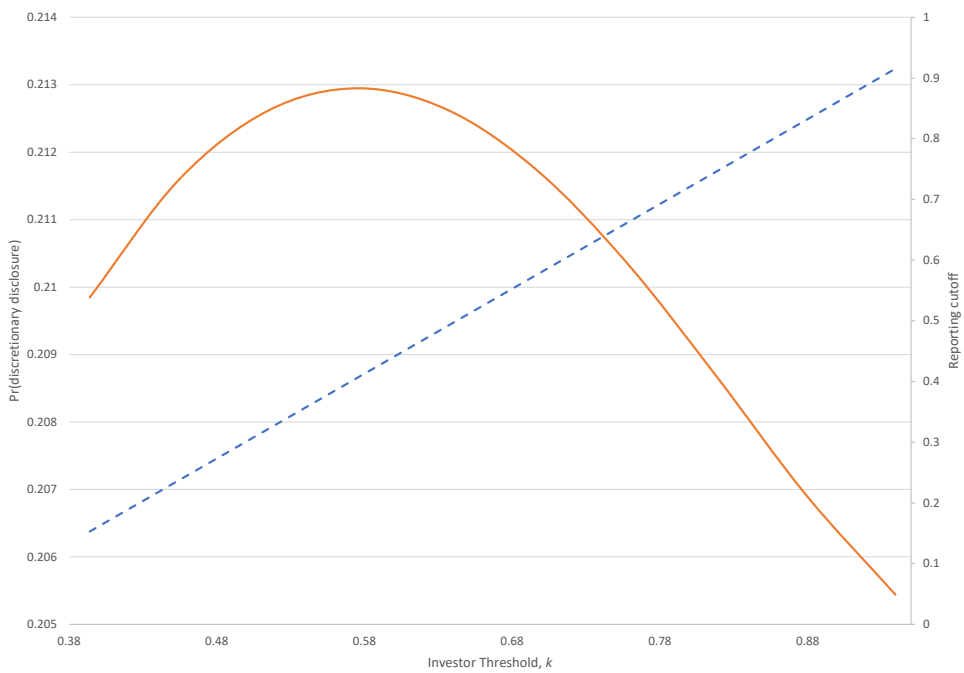

Figure 3: Probability of discretionary disclosure and reporting cutoff, $\widehat{\omega}$, for various values of sufficiently high k, with with $\omega \sim \text{Beta}\,(1,2)$, $p = 0.4$, and an optimal binary reporting system. The solid line is the probability of discretionary disclosure, and the dashed line is the reporting cutoff, $\widehat{\omega}$.

reports whenever $E[\omega] > k$.[18] Let $\widehat{\omega}^0 \equiv \lim_{p\to 0} \widehat{\omega}$. Note that $\widehat{\omega}^0 = \min_p \widehat{\omega}$.

With mandatory disclosure, no disclosure implies that the firm did not observe private information. The firm communicates the true underlying state with probability p, and gets the non-disclosure payoff with probability $1-p$. Consider the case of high k. The firm's expected payoff is:

$$p\left(E\left[\omega\right] - CF(k)\right) + (1-p)\left(E\left[\omega\right] - CF\left(\widehat{\omega}^0\right)\right) = E[\omega] - C\left(pF(k) + (1-p)F\left(\widehat{\omega}^0\right)\right).$$

Recall that the expected payoff under discretionary disclosure is $E[\omega] - C \times F\left[\widehat{\omega}\right]$ as shown in Proposition 1, part (ii). The difference between the expected payoffs is

$$\Delta \equiv C\left(F(\widehat{\omega}) - pF(k) - (1-p)F(\widehat{\omega}^0)\right).$$

[18]Note that $E[\omega] < k$ is a more demanding condition than $\widehat{k} < k$. To see why, recall that by Definition 1 the cutoff $\widehat{k}$ is uniquely defined by $\int_{\underline{\omega}}^{\overline{\omega}} \omega f(\omega) d\omega - \left(\widehat{k} + \frac{p}{1-p}\int_{\underline{\omega}}^{\widehat{k}} F(\omega) d\omega\right) = 0$, which implies $\widehat{k} < E[\omega]$.

By Proposition 1 and Corollary 1, for any $p > 0$, we have $\widehat{\omega} \in (\widehat{\omega}^0, k)$, which implies

$$\Delta \propto \underbrace{(F(\widehat{\omega}) - F(\widehat{\omega}^0))}_{>0} - p \underbrace{(F(k) - F(\widehat{\omega}^0))}_{>F(\widehat{\omega})-F(\widehat{\omega}^0))>0}.$$

From the expression above, it is ex ante unclear whether the firm is better or worse off with mandatory disclosure because the comparison depends on the properties of $F(\cdot)$ and the magnitude of p and k. Our next result clarifies.

Proposition 2 *The firm prefers a discretionary disclosure regime.*

The discussion preceding our result focuses on the case of high k while Proposition 2 speaks to all possible values of k. To see why this result also holds for low k, note that with a discretionary disclosure regime, the firm always meets the threshold because it has the option to withhold low signals that would have lead to missing the threshold. In contrast, under a mandatory regime, the firm is obliged to disclose any observed signal. To summarize: in a discretionary disclosure regime, the firm can choose whether to disclose or withhold signals, and this option may be valuable. However, it makes nondisclosure worse because uninformed firms are pooled with firms that privately observed low states. In a mandatory disclosure regime, the firm lacks the option to withhold, conditional on observing the private signal, but is not penalized as harshly following nondisclosure. Overall, the option to withhold signals provides a greater benefit to the firm in expectation.[19]

5.2. Alternative objective functions

We discuss alternative objective functions, highlighting along the way our motivation for the specification of the firm's objective in our main analysis. Our motivating examples (e.g., raising debt, delisting, index inclusion, bank runs, and rating downgrades) exhibit several properties of our model. Each provides a nonlinearity in the firm's objective function that satisfies the general conditions set forth by Kamenica and Gentzkow (2011) for a sender to benefit from the ability to design a reporting system that can influence a receiver's decisions.

[19] Versano (2019) shows that voluntary disclosure may be socially desirable when the firm's disclosure is not always verifiable.

5.2.1. A linear or convex objective function

An objective function that is purely linear in the investor's posterior beliefs, such as maximizing the posterior expectation, would leave the firm indifferent across reporting systems (by the law of iterated expectations) and result in a disclosure strategy as described in Lemma 1 following any report. A convex objective function, such as option-based compensation, would result in a perfectly informative reporting system, leaving the disclosure stage a moot issue.[20]

5.2.2. Jump-only objective and a no-disclosure equilibrium

Technically, a discrete jump (or drop) in the firm's expected payoff by itself (i.e., $\pi(r, m, \boldsymbol{\beta}) = -C\mathbb{1}_{E[\omega|r,m,\boldsymbol{\beta}]<k}$) would also result in an imperfectly informative optimal reporting system that depends on the properties of subsequent disclosures and the prior beliefs. However, an objective consisting only of a jump (without the linear part) would have flat regions that would leave the firm potentially indifferent between disclosing and withholding private signals.

Proposition 3 *If the firm's objective is only to meet the investor's threshold, there exists an equilibrium in which the firm never discloses its private signal and the reporting system is set as in the reporting benchmark.*

Without the linear portion of the objective function, there is an equilibrium in which the firm credibly commits not to disclose private signals by choosing a reporting system equivalent to the one it would choose if private signals were not available, i.e., under the reporting benchmark as $p \to 0$. This strategy would be preferred by the firm in the absence of the linear component, as can be observed by the fact that the firm's expected payoff is weakly decreasing in p, via Corollary 2 and Proposition 1. Furthermore, this strategy would yield a reporting system that is independent of the informational properties of the firm's private signals, eliminating the interesting interdependencies between reporting system design and

[20] With a convex objective, in the reporting benchmark, the firm would set a perfectly informative reporting system (Kamenica and Gentzkow 2011). This represents the best the firm can do in the full game, as any distribution of beliefs that could be induced by the reporting system and subsequent discretionary disclosures jointly could also be induced by the reporting system alone.

discretionary disclosure.[21]

Although we find this result interesting, the assumption of no further benefit to disclosure of a high signal is unlikely to be descriptive of situations that firms actually face given the many other roles that have been ascribed to discretionary disclosure (e.g., increasing stock price). Any marginal benefit to disclosure of high private signals beyond that of meeting a crucial threshold, in and of itself, no matter how negligible, suffices to eliminate this equilibrium, notwithstanding that the firm may be better off with no disclosure.

5.2.3. Shallower (or flat) objective for below-threshold posterior beliefs

In this subsection, we consider an alternative in which the firm's objective is shallower or even flat for $E[\omega|r, m, \boldsymbol{\beta}] < k$. One example of such an objective corresponds to the firm's payoff if a lender uses a 'live-or-die' contract rather than standard debt, as discussed in Innes (1993).[22] The difference in slopes complicates the analysis because the law of iterated expectations can no longer be exploited to simplify the firm's objective.[23] Nonetheless, as we show below, the results from our main analysis hold as long as the firm faces a sufficiently high cost for missing the investor's threshold.

Let the alternative objective be defined by

$$\pi^o(r, m, \boldsymbol{\beta}) = E[\omega|r, m, \boldsymbol{\beta}] - (C + \delta E[\omega|r, m, \boldsymbol{\beta}])\, \mathbb{1}_{E[\omega|r,m,\boldsymbol{\beta}]<k} \tag{8}$$

with $\delta \in [0, 1]$. This payoff function is graphically illustrated in Figure 4. For $\delta = 0$, we have our original objective. Taking $\delta = 1$ could represent, for example, a firm that shuts down if posterior expectations are below k.

Proposition 4 *Let* $C^o \equiv \delta \frac{F(\widehat{\omega})E[\omega|\omega<\widehat{\omega}]-F(k)E[\omega|\omega<k]}{F(k)-F(\widehat{\omega})}$. *When the firm's objective is as defined*

[21] Any choice of $\boldsymbol{\beta}$ other than those optimal for the reporting benchmark case would indicate that the firm intended to make use of private information subsequently received. In this sense, the choice of $\boldsymbol{\beta}$ serves as an indicator to the investor that private information will not be disclosed.

[22] Innes (1993) shows that the 'live-or-die' financing contract, which in our setting is ω for $\omega < k$ and 0 for $\omega > k$, emerges as an optimal solution in a setting with a risk-neutral capital market with information asymmetry between entrepreneurial borrowers and their investors. When a monotonicity constraint is imposed, Innes (1993) recovers a standard debt contract.

[23] The additively separable linear component of the firm's objective function used in our main analysis has appealing analytical features. Through the law of iterated expectations, the firm's expected payoff from the linear component is independent of the reporting system because the expected value of the posterior expectation is equal to the prior expectation.

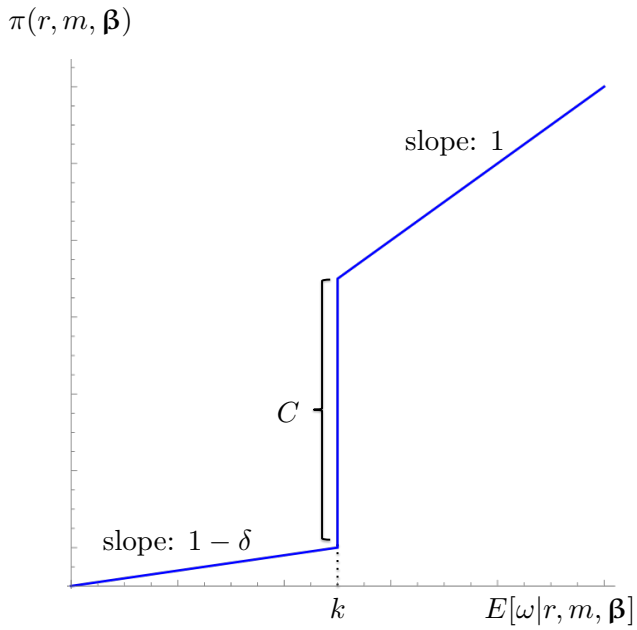

Figure 4: Firm's payoff with shallower objective for posterior beliefs below k

in (8)*, the optimal reporting system is as described in Proposition 1 if either k is low or $C \geq C^o$. If k is high and $C < C^o$, then the optimal reporting system is perfectly informative.*

The overall effect of $\delta > 0$ is to make the firm's payoff more convex, and convexity encourages the firm to set a perfectly informative reporting system. For low k, flattening the portion of the objective before k should have no effect, as the firm continues to avoid the threshold-based cost following uninformative reports and nondisclosure. For high k and $C > C^o$, the firm's objective is not too convex, and the solution in Proposition 1 remains optimal. For high k and $C < C^o$, the firm's objective is sufficiently convex, such that the optimal reporting system is perfectly informative and discretionary disclosures provide no additional information. Overall, our results in this section imply that our main results are robust to allowing the alternative objective function described by π^o in equation (8), as long as either k is low or $C > C^o$.[24]

As for the discretionary disclosure stage, as long as $\delta < 1$, our analysis for the disclosure stage will not change qualitatively, because it relies only on the firm's objective increasing everywhere. However, if $\delta = 1$, the firm's objective is flat for posterior expectations below k. Recall that for optimal reporting systems, the disclosure cutoff is below k only if k is high

[24]For $\delta = 0$, as in our main analysis, $C > C^o \Leftrightarrow C > 0$, which is always satisfied.

and a report $r_j \neq r_g$ is produced. In such a case, with $\delta = 1$ the firm will be indifferent between disclosure and nondisclosure, because $r_j \neq r_g$ implies that the state is below k and the objective for posterior expectations below k is flat.[25] Note that the disclosure strategy after a good report remains the same as described in our main analysis. Furthermore, if k is low, the disclosure cutoff following any report (even an uninformative one) is above k, implying that the flat region is irrelevant. In both of these cases (r_g for high k or any report for low k) the optimal conditional disclosure strategy is described by Lemma 1.

5.3. Endogenous investor threshold

In this section we explore two ways of endogenizing the investor's threshold. We allow the threshold to emerge from an agency issue or be set by a regulator who seeks to glean information from the firm.

5.3.1. Pledgeable assets and agency costs

The investor threshold, k, can be motivated from the well-studied agency problem in Holmström and Tirole (1997). Adopting notation from Göx and Wagenhofer (2009), we assume that the firm seeks a loan of $I > 0$ from a risk neutral lender to finance a new project and has existing assets that it can pledge as collateral. Let ω be the value of the assets. The assets are more valuable to the firm than to the lender, who values them at $\gamma\omega$, where $\gamma \in (0, 1)$ is a factor that represents a collateral liquidation discount. The new project pays X if it succeeds and 0 if it fails. It succeeds with probability q_H if the firm's owner-manager works hard and $q_H - \Delta q$ otherwise.[26] Working hard costs the manager ν. The loan has a face value of d_S, such that it pays d_S if the project succeeds and 0 if the project fails. Upon failure, though, the lender gets $\gamma\omega$ from liquidating the firm's assets. The owner-manager will work hard if the incentive compatibility constraint, $X - d_S + A - \nu/\Delta q \geq 0$, is met. The lender will provide a loan as long as its participation constraint, $q_H d_S + (1 - q_H)\gamma\omega - I \geq 0$, is met.

[25] Even with an impairment-type reporting system the firm is indifferent between disclosure and nondisclosure after $r_j \neq r_g$ because the reporting system already perfectly reveals all states below the reporting cutoff $\widehat{\omega}$.

[26] To maintain the linear portion of the overall objective, π, we must also assume that there are minority investors who value the firm and that the owner-manager benefits from them assigning higher valuations to the firm's asset in place as well.

The value of the loan to the firm is $q_H(X - d_s) - (1 - q_H)\omega$, which we assume is positive for all possible values of ω.

As in Göx and Wagenhofer (2009), the lender will provide the loan and the manager will work hard if the firm pledges assets of $\omega = \hat{A} \equiv \frac{I - q_H(X - \nu/\Delta q)}{q_H + (1 - q_H)\gamma}$ and the debt has a face value of $d_S = X - \frac{\nu}{\Delta q} + A$. The lender will believe the firm has sufficient assets to pledge if $E[\omega|r, m] \geq \hat{A}$. The features of our setting are thus well represented by taking the investor/lender's threshold as $k = \hat{A}$ and the cost to the firm of missing the threshold in posterior beliefs as equal to the cost of not obtaining the loan, $C = q_H(X - d_s) - (1 - q_H)\omega$. Corollary 1 shows that the reporting cutoff, $\widehat{\omega}$ is increasing in k when k is taken as an exogenous parameter. If k is a result of the agency problems described in Göx and Wagenhofer (2009), then the reporting cutoff will also be increasing in the severity of the effort motivation problem, $\nu/\Delta q$, increasing in the loss from liquidating assets, γ, and decreasing in the new project's expected profitability, $q_H X - I$.[27]

5.3.2. Threshold chosen by an information-motivated investor

In this section, we allow the threshold, k, to be optimally chosen by an investor, information intermediary, or regulator to maximize the probability that they or their constituents learn the state. This could be a board designing a discretionary bonus threshold or an exchange designing a price-based listing requirement, both with an eye toward motivating the potentially informed party (i.e., the manager or firm) to be more forthcoming. In the former case, a more forthcoming manager might allow the board to make better decisions, while in the latter case a more forthcoming firm might have better liquidity on the exchange. The threshold designer's problem is: $\max_k \Pr(Var(\omega|r, m, \boldsymbol{\beta}) = 0)$.[28] To facilitate analysis and heighten the conflict between the threshold designer and the firm, we assume that the firm bears a (possibly vanishing) cost from making the reporting system more informative. This cost will cause the firm to choose the least informative reporting system from the set of reporting systems satisfying the conditions in Proposition 1.

[27] These results correspond to Corollary 2 of Göx and Wagenhofer (2009).

[28] We use the probability of knowing the state with certainty as our measure of the information content of reports and messages, jointly. With general distributions, other measures of information content (e.g., entropy reduction and the variance of conditional expectations) are not tractable.

Proposition 5 *If the firm bears a (possibly vanishing) cost from making the reporting system more informative, then a threshold designer choosing k to maximize the probability of learning the underlying state with certainty weakly prefers an interior $k < \overline{\omega}$.*

If $k > \widehat{k}$, the firm's preferred optimal reporting system is a binary one that reports r_g if $\omega > \widehat{\omega}$ and r_b otherwise. If $k \leq \widehat{k}$, the firm's preferred optimal reporting systems is a completely uninformative one. Under both of these systems, Pr(state ω is known) = Pr(disclosure). Section 4.1 explains that the probability of disclosure under an uninformative reporting system is independent of k. Under a binary reporting system, the probability of disclosure is non-monotonic in k (see Corollary 5). Note that if $k \to \overline{\omega}$ then the least informative system is a binary one that maps $\overline{\omega}$ into report r_g and all states $\omega_i < \overline{\omega}$ to r_b. Such a system is essentially uninformative in the limiting case, so the threshold designer will be indifferent between $k = \overline{\omega}$ and any $k < \widehat{k}$. Nonetheless, there may be an interior $k \in (\widehat{k}, \overline{\omega})$ that yields a strictly higher probability of learning the underlying state, which would cause the threshold designer to have a strict preference for that k.

The result in Proposition 5 speaks to the potential choice of contract terms or investment rules. We can, for instance, interpret the choice of k as an institutional investor's rule for choosing which bonds to hold. Although this choice is motivated by concerns outside our model (e.g., trading off interest income and credit risk), our result suggests that a rule such as "hold investment-grade credits" may be consistent with reducing the probability of ex post uncertainty as well.

This result further points to the importance of the interaction between discretionary disclosure and reporting system design. In the disclosure benchmark, k does not affect the probability of disclosure, so the regulator would be indifferent across all values of k. The interaction between reporting system design and subsequent disclosure can yield a strict preference for an interior k because the firm adjusts the design of the reporting system in response to changes in k.

5.4. Manipulation of the report

We now consider a setting in which the firm can opportunistically manipulate reports before they are transmitted to the representative investor. Specifically, we assume that with exoge-

nous probability, $q_m \in [0,1]$, the firm can manipulate the realized report, r, into any other potential report. It could be that the firm gets the opportunity to manipulate the report with probability q_m (e.g., depending on the firm's operations). Alternatively, firm might try to manipulate every report upward but only succeeds with probability q_m (e.g., because of auditor scrutiny). Following the release of the potentially manipulated report, r, the firm plays the discretionary disclosure subgame is in the main model.

The firm's payoff is increasing in the investor's posterior expectation of the state. Because we assume that a manipulated report can become *any* other report, the firm will always manipulate the report that induces the highest possible posterior expectation (i.e., the highest report). As a result, the highest report will be transmitted to investors more frequently and lower reports will be transmitted less frequently. While simplistic, this concisely captures the central feature of opportunistic report manipulation.

If the investor's threshold, k, is low, this manipulation technology will not affect the firm's expected payoff. Define $E\omega_r^{\varnothing} \equiv E[\omega|r, m = \varnothing]$ and denote the highest and lowest possible reports by $\overline{r}$ and $\underline{r}$, respectively. Note that the condition in Proposition 1, part (i), must be satisfied for $\underline{r}$ before manipulation, implying that it is slack for any other report. Introducing manipulation will tighten the condition for the highest possible report, but it will remain satisfied because $E\omega_{\overline{r}}^{\varnothing}$ with manipulation is a convex combination of its unmanipulated value and the unmanipulated values of $E\omega_r^{\varnothing}$ for other reports. Since all of these $E\omega_r^{\varnothing}$ are greater than the investor threshold, $E\omega_{\overline{r}}^{\varnothing}$ with manipulation will be as well. Furthermore, although manipulation lowers the probability of other reports, it does not change their information content, i.e., $E\omega_r^{\varnothing}$ is not affected by manipulation for $r \neq \overline{r}$. Hence, the condition in Proposition 1, part (i), will continue to be satisfied for all reports, and the firm's expected payoff will be maintained at $E[\omega]$ when k is low and probabilistic manipulation is introduced.

If the investor's threshold, k, is high, the potential to manipulate with probability q_m is more problematic. Manipulation will cause the good report, r_g, to be generated with a higher frequency, i.e., $q_m F(\widehat{\omega}) + 1 - F(\widehat{\omega})$ rather than $1 - F(\widehat{\omega})$. This will cause the posterior expectation following a good report and nondisclosure to fall below the investor's threshold. If q_m is low, the firm can maintain the condition in Proposition 1, part (ii), that $E[\omega|r_g, m = \varnothing] = k$, by raising the reporting cutoff, $\widehat{\omega}$, such that the firm continues to

avoid the cost of missing the investor's threshold whenever the good report is transmitted. Institutionally, this might be viewed as an increase in an impairment cutoff, consistent with a more conservative reporting system. Bayesian plausibility implies that, if the condition holds both with and without manipulation, then the probability of transmitting the good report with manipulation (and a higher reporting cutoff) must equal the probability of transmitting the good report without manipulation (and a lower reporting cutoff). Hence, the firm's expected payoff will not be affected by manipulation when q_m is low.

If q_m is high, however, it may be impossible for the firm to set a reporting system that satisfies the condition in Proposition 1, part (ii). At the upper limit of $q_m = 1$, the firm will always report r_g. Investors disregard the uninformative r_g, and the firm's expected payoff falls below what could be achieved without manipulation.[29] With high k, therefore, the potential to manipulate makes the firm weakly worse off in expectation.

Our finding that probabilistic manipulation makes the firm weakly worse off is not surprising, given that the firm could have baked the probabilistic manipulation into the reporting system. The firm is no worse off if it can undo the effects of the manipulation technology and maintain the same distribution of beliefs, as with low q_m, but is strictly worse off if the reporting system cannot be adjusted to undo these effects, as with high q_m.

6. Conclusion

This paper studies the interactions between reporting system design and discretionary disclosure in a model where a firm employs both channels to influence investors' beliefs about the firm's state. In our model, the firm benefits from investors holding more positive beliefs in general and incurs a cost from investors having beliefs that are below a certain threshold. Such thresholds are ubiquitous in financial markets, liquidation/continuation choices, listing requirements, index inclusion, credit ratings, analyst recommendations, and stress tests. We derive the firm's disclosure strategies and find that the set of optimal reporting systems is generally large. When the threshold is high, both binary and impairment-type reporting

[29] The firm's expected payoff with an uninformative reporting system when k is high is $E[\omega] - C(1 - p(1 - F(k)))$, which is strictly lower than the firm's expected payoff absent manipulation, $E[\omega] - CF(\widehat{\omega})$. With $q_m = 1$, the firm only avoids the cost of missing the investor's threshold when it discloses a private signal greater than k, which happens with probability $p(1 - F(k))$.

systems are in the set of optima. When the threshold is low, an uninformative reporting system is in the optimal set.

Our results contribute to the understanding of interdependencies between the properties of reporting systems implemented by firms and their discretionary disclosure choices. These interactions are important both because they are two central means firms use to influence stakeholders and because the interdependencies between them can lead to incorrect inferences from empirically observed associations. We show how the presence of a disclosure stage can alter the firm's reporting system design. At an institutional level our results speak to the properties of various accounting methods and disclosure policies that we observe in practice. Our findings suggest, for example, that firms more likely to be privately informed about asset values may set higher impairment cutoffs.

There are several variations on the structure we impose that future research might address. Our model is static with two stages the order of which could be reversed and further stages added to explore dynamic interdependencies. We focus on a friction in the form of a threshold on investors' posterior beliefs that if unmet implies a loss in expected utility, and discretionary disclosures, if made, are freely verifiable. Alternatives include reports or disclosures subject to direct costs or lacking in credibility.

Appendix: Notation and Proofs

Variable	Definition
ω	State variable (e.g., collateral value)
k	Investor's threshold
$F(\omega)$	Prior cumulative distribution function (CDF) of the state variable
$f(\omega)$	Prior probability density function (PDF) of the state variable
r	Report made by firm
$\boldsymbol{\beta}$	Parameters governing the reporting system
s	Private signal potentially observed by firm
p	Probability with which firm observes private signal, s
m	Verifiable message sent by firm based on private signal, s
C	Cost to firm of missing investor's threshold, i.e., if $E\left[\omega \vert r, m, \boldsymbol{\beta}\right] < k$

Table A.1: Notation for main model

Proof of Lemma 1: When deciding whether to disclose the observed signal realization s, the firm compares $\pi(r, m = s, \boldsymbol{\beta})$ with $\pi(r, m = \varnothing, \boldsymbol{\beta})$ and chooses the message that yields a higher payoff. Note that $\pi(\cdot)$ is a one-to-one function. Hence, $\pi(x) \geq \pi(y) \Leftrightarrow x \geq y$. In our setting this implies the firm discloses if and only if

$$E[\omega|r, m = s, \boldsymbol{\beta}] \geq E[\omega|r, m = \varnothing, \boldsymbol{\beta}].$$

This is similar to the disclosure cutoff condition in DJK, but with their equality replaced by a weak inequality. With the disclosure cutoff $t_{r_g,\boldsymbol{\beta}}$ defined as the lowest signal that the firm discloses rather than withholds following a report r_j with a reporting system $\boldsymbol{\beta}$, we have,

$$t_{r_j,\boldsymbol{\beta}} \in \arg\min_t \quad t \tag{A.1}$$

$$\text{s.t. } t \geq \frac{(1-p)E_{r_j}\left[\omega|r_j, s = \varnothing, \boldsymbol{\beta}\right] + pE_{r_j}\left[\omega|r_j, s < t, \boldsymbol{\beta}\right]}{1 - p + pF_{r_j,\boldsymbol{\beta}}(t)} \tag{A.2}$$

We can write equation (A.2) using Lebesgue-Stieltjes integrals as

$$t \geq \frac{(1-p)\int_{\underline{\omega}_{r_j}}^{\overline{\omega}_{r_j}} \omega dF_{r_j,\boldsymbol{\beta}}(\omega) + p\int_{\underline{\omega}_{r_j}}^{t} \omega dF_{r_j,\boldsymbol{\beta}}(\omega)}{1 - p + pF_{r_j,\boldsymbol{\beta}}(t)}. \tag{A.3}$$

Integration by parts yields

$$\begin{aligned}\int_{\underline{\omega}_{r_j}}^{t} \omega dF_{r_j,\boldsymbol{\beta}}(\omega) &= t\left(\lim_{\omega \searrow t} F_{r_j}(\omega)\right) - \underline{\omega}_{r_j}\left(\lim_{\omega \nearrow \underline{\omega}_{r_j}} F_{r_j}(\omega)\right) - \int_{\underline{\omega}_{r_j}}^{t} F_{r_j,\boldsymbol{\beta}}(\omega)d(\omega) \\ &= tF_{r_j}(t) - \int_{\underline{\omega}_{r_j}}^{t} F_{r_j,\boldsymbol{\beta}}(\omega)d(\omega)\end{aligned}$$

which we can substitute back in to inequality (A.3) and rearrange to get

$$\int_{\underline{\omega}_{r_j}}^{\overline{\omega}_{r_j}} \omega dF_{r_j,\boldsymbol{\beta}}(\omega)d\omega \leq t + \frac{p}{1-p}\int_{\underline{\omega}_{r_j}}^{t} F_{r_j,\boldsymbol{\beta}}(\omega)d\omega. \tag{A.4}$$

The left-hand side is independent of t, while the right-hand side is strictly increasing in t. For $t = \underline{\omega}_{r_j}$, the inequality is violated, and for $t = \overline{\omega}_{r_j}$ the inequality is slack. As t is drawn from the $\left[\underline{\omega}_{r_j}, \overline{\omega}_{r_j}\right]$ interval, there exists a minimum t such that the inequality in (A.4) is satisfied.

If $F_{r_j,\boldsymbol{\beta}}(\omega)$ is differentiable, then $f_{r_j,\boldsymbol{\beta}}(\omega) = F'_{r_j,\boldsymbol{\beta}}(\omega)$ exists and the inequality in (A.4) can be replaced with an equality, yielding a unique value for t, as in DJK. If $F_{r_j,\boldsymbol{\beta}}(\omega)$ is not differentiable in $[\underline{\omega}_{r_j}, \overline{\omega}_{r_j}]$, then there are three possibilities:

1. If $F_{r_j,\boldsymbol{\beta}}(t)$ is strictly increasing and continuous around the t that satisfies (A.4) with equality, then $F_{r_j,\boldsymbol{\beta}}(t)$ is locally differentiably and that t uniquely defines $t_{r_j,\boldsymbol{\beta}}$. This occurs if the distribution of ω is locally continuous around $t_{r_j,\boldsymbol{\beta}}$.

2. If $F_{r_j,\boldsymbol{\beta}}(t)$ is increasing discontinuously at the t that satisfies (A.4), then that t again

uniquely defines $t_{r_j,\boldsymbol{\beta}}$. This occurs if the distribution of ω is discrete around $t_{r_j,\boldsymbol{\beta}}$ and there is nonzero probability that $\omega = t_{r_j,\boldsymbol{\beta}}$ following report r_j under reporting system $\boldsymbol{\beta}$.

3. The conditional CDF at $t_{r_j,\boldsymbol{\beta}}$ can be flat if $t_{r_j,\boldsymbol{\beta}}$ falls in a region where the conditional probability of ω is zero. Specifically, we can have $dF_{r_j,\boldsymbol{\beta}}(t_{r_j,\boldsymbol{\beta}}) = 0$. Define $\widehat{t}_{r_j,\boldsymbol{\beta}}$ as the next highest ω with positive probability mass, i.e., $\widehat{t}_{r_j,\boldsymbol{\beta}} = \min_\omega \text{s.t.} dF_{r_j,\boldsymbol{\beta}}(t_{r_j,\boldsymbol{\beta}}) > 0$ and $\omega > t_{r_j,\boldsymbol{\beta}}$. The firm will be indifferent between disclosure thresholds in $[t_{r_j,\boldsymbol{\beta}}, \widehat{t}_{r_j,\boldsymbol{\beta}})$, but setting the disclosure threshold in this region implies no change in disclosure strategy for private signals with positive probability.

Proof of Proposition 1: The proof of item (i) follows from the discussion in the text. Here, we only prove item (ii). In the following, we first conjecture a reporting system and then prove that it is at least weakly preferred by the firm. Our conjectured optimal reporting system, $\boldsymbol{\beta}^*$, generates $r = r_g$ with probability $\beta_i^g = 1$, for all $\omega_i \geq \widehat{\omega}$. It is obvious that for any report realization $r_i \neq r_g$ the investor's threshold is not met. To ensure that the threshold is always met after a good report, i.e., that $E[\omega|r = r_g, m, \boldsymbol{\beta}^*] \geq k$ for all $m = \{\varnothing, s\}$, it has to be the case that, upon realization of $r = r_g$ and observing $s = k$, the firm is indifferent between disclosing and withholding, i.e.,

$$\pi(E[\omega|r = r_g, m = \varnothing, \boldsymbol{\beta}^*]) \quad = \quad \pi(E[\omega|r = r_g, m = k, \boldsymbol{\beta}^*]). \tag{A.5}$$

This implies that $E[\omega|r = r_g, m = \varnothing, \boldsymbol{\beta}^*] = E[\omega|r = r_g, m = k, \boldsymbol{\beta}^*]$ because $\pi(\cdot)$ is a one-to-one function. Because messages are verifiable and private signals, if observed, reveal the state, we have $E[\omega|r = r_g, m = k, \boldsymbol{\beta}^*] = k$. Therefore, condition (A.5) is equivalent to

$$E[\omega|r = r_g, m = \varnothing, \boldsymbol{\beta}^*] \quad = \quad k$$

and implies that, after a good report, $E[\omega|r = r_g, m, \boldsymbol{\beta}^*] \geq k$ for all $m = \{\varnothing, s\}$. This is so because: (i) for any $s \geq k$ the firm will disclose and so $E[\omega|r = r_g, m = s, \boldsymbol{\beta}^*] = s > k$; (ii) for any $s < k$ the firm will withhold and so $E[\omega|r = r_g, m = \varnothing, \boldsymbol{\beta}^*] = k$.

Now consider the reporting cutoff $\widehat{\omega}$. It is set to maximize the probability of sending r_g subject to $E[\omega|r = r_g, m, \boldsymbol{\beta}^*] \geq k$, i.e., to maximize the probability of achieving a posterior belief above the threshold k. The probability of sending r_g conditional on sending it for any $\omega \geq \widehat{\omega}$ is $1 - F_{r_g,\boldsymbol{\beta}^*}(\widehat{\omega})$. Maximizing $1 - F_{r_g,\boldsymbol{\beta}^*}(\widehat{\omega})$ is equivalent to minimizing $\widehat{\omega}$ because CDFs are increasing. The firm's problem can be written as

$$\begin{aligned} \min_{\omega \in [\underline{\omega}, k]} \quad & \widehat{\omega} \\ \text{s.t.} \quad & E[\omega|r_g, \boldsymbol{\beta}^*] = k + \frac{p}{1-p} \int_{\widehat{\omega}}^{k} F_{r_g,\boldsymbol{\beta}^*}(\omega)\, d\omega. \\ & \beta_i^g = 1 \quad \forall \quad \omega_i \geq \widehat{\omega} \end{aligned} \tag{A.6}$$

Here condition (A.6) comes from substituting $t_{r_g,\boldsymbol{\beta}^*} = k$ into equation (1). We can use equation (1) because $F_{r_g,\boldsymbol{\beta}^*}(\omega)$ is continuous and differentiable, due to the fact that $F(\omega)$ is differentiable and r_g induces a distribution $F_{r_g,\boldsymbol{\beta}^*}(\omega)$ with connected support on $[\widehat{\omega}, \overline{\omega}]$.

Note that the conditional PDF and CDF are defined as functions of the unconditional PDF and CDF, respectively, as $f_{r_g,\boldsymbol{\beta}^*}(\omega) = \frac{f(\omega)}{F(\overline{\omega})-F(\widehat{\omega})} = \frac{f(\omega)}{1-F(\widehat{\omega})}$ and $F_{r_g,\boldsymbol{\beta}^*}(\omega) = \frac{F(\omega)-F(\widehat{\omega})}{F(\overline{\omega})-F(\widehat{\omega})} = \frac{F(\omega)-F(\widehat{\omega})}{1-F(\widehat{\omega})}$, so we can restate condition (A.6) as

$$\int_{\widehat{\omega}}^{\overline{\omega}} \omega f(\omega)\, d\omega - (1 - F(\widehat{\omega}))\, k - \frac{p}{1-p}\int_{\widehat{\omega}}^{k} F(\omega)\, d\omega + \frac{p}{1-p} F(\widehat{\omega})\,(k - \widehat{\omega}) = 0 \tag{A.7}$$

and define $G(\widehat{\omega})$ as the left-hand side (LHS) of equation (A.7). Note that

$$\begin{aligned}
\frac{dG(\widehat{\omega})}{d\widehat{\omega}} &= -\widehat{\omega} f(\widehat{\omega}) + f(\widehat{\omega})\, k + \frac{p}{1-p} F(\widehat{\omega}) + \frac{p}{1-p} f(\widehat{\omega})\,(k - \widehat{\omega}) - \frac{p}{1-p} F(\widehat{\omega}) \\
&= f(\widehat{\omega})\,(k - \widehat{\omega}) \left(\frac{1}{1-p}\right) \\
&\propto k - \widehat{\omega}.
\end{aligned}$$

Hence, $G(\widehat{\omega})$ is increasing if $k > \widehat{\omega}$ and decreasing otherwise. Furthermore,

$$\begin{aligned}
\lim_{\widehat{\omega}\to k} G(\widehat{\omega}) &= \int_{k}^{\overline{\omega}} \omega f(\omega)\, d\omega - (1 - F(k))\, k \\
&= \overline{\omega} - k - \int_{k}^{\overline{\omega}} F(\omega)\, d\omega \\
&= \int_{k}^{\overline{\omega}} (1 - F(\omega))\, d\omega > 0, \text{ and} \qquad \text{(A.8)} \\
\lim_{\widehat{\omega}\to \underline{\omega}} G(\widehat{\omega}) &= \int_{\underline{\omega}}^{\overline{\omega}} \omega f(\omega) d\omega - \left(k + \frac{p}{1-p}\int_{\underline{\omega}}^{k} F(\omega) d\omega\right). \qquad \text{(A.9)}
\end{aligned}$$

Note that (A.9) is negative because $k > \widehat{k}$. The Intermediate Value Theorem implies that $G(\widehat{\omega}) = 0$ has a solution, $\widehat{\omega} \in (\underline{\omega}, k)$. $G(\cdot)$ is strictly increasing in this range, so $\widehat{\omega}$ is the unique solution in $(\underline{\omega}, k)$.

With $\boldsymbol{\beta}^*$, the firm avoids the cost, C, whenever the state is above $\widehat{\omega}$ and incurs it otherwise. Thus, the firm's expected payoff is $E[\pi] = E[\omega] - C \times F(\widehat{\omega})$.

We now prove that the conjectured reporting system, $\boldsymbol{\beta}^*$, is an optimum by showing that any alternative reporting system, $\boldsymbol{\beta}'$, provides a lower expected payoff. We first consider three specific ways in which $\boldsymbol{\beta}'$ can differ from $\boldsymbol{\beta}^*$.

(i) If $\boldsymbol{\beta}'$ differs from $\boldsymbol{\beta}^*$ only in that it produces r_g with positive probability for some state(s) lower than $\widehat{\omega}$, then Bayesian plausibility implies that $E[\omega|r_g, \boldsymbol{\beta}'] < E[\omega|r_g, \boldsymbol{\beta}^*]$, which in turn implies that $E[\omega|r_g, m = \varnothing, \boldsymbol{\beta}'] < k$. Therefore, the firm will only avoid the cost following a disclosure of $m \geq k$ and its expected payoff will be $E[\pi|\boldsymbol{\beta}'] = E[\omega] - C \times (1 - p(1 - F(k)))$. Now,

$$E[\pi|\boldsymbol{\beta}'] < E[\pi|\boldsymbol{\beta}^*] \Leftrightarrow (1 - p(1 - F(k)) - F(\widehat{\omega}) > 0.$$

Integration by parts and algebraic manipulation of equation (A.7) yields

$$(1-p(1-F(k))-F(\widehat{\omega}) \quad = \quad \frac{1}{k}\left((1-p)\int_{\widehat{\omega}}^{\overline{\omega}} \omega f(\omega)\, d\omega + p\int_{\widehat{\omega}}^{k} \omega f(\omega)\, d\omega\right) > 0,$$

which implies $E[\pi|\boldsymbol{\beta}'] < E[\pi|\boldsymbol{\beta}^*]$.

(ii) If $\boldsymbol{\beta}'$ differs from $\boldsymbol{\beta}^*$ only in that it produces reports other than r_g with positive probability for some set of states with elements $\omega_i^\dagger \in (\widehat{\omega}, k]$, then $E[\omega|r_g, m=\varnothing, \boldsymbol{\beta}'] \geq k$, but $\Pr(r_g|\boldsymbol{\beta}') < \Pr(r_g|\boldsymbol{\beta}^*)$, so the firm will bear the cost more frequently and have a lower expected payoff.

(iii) If $\boldsymbol{\beta}'$ differs from $\boldsymbol{\beta}^*$ only in that it produces reports $r_j \neq r_g$ with positive probability for some set of states with elements $\omega_i^\dagger \in (k, \overline{\omega}]$, then $E[\omega|r_g, m=\varnothing, \boldsymbol{\beta}'] < k$. If $E[\omega|r_j, m=\varnothing, \boldsymbol{\beta}'] < k$, then the firm will only avoid the cost following a disclosure of $m \geq k$ and its expected payoff will be $E[\pi|\boldsymbol{\beta}'] < E[\pi|\boldsymbol{\beta}^*]$, as show in item (i). If $E[\omega|r_j, m=\varnothing, \boldsymbol{\beta}'] \geq k$, then the total effect is similar to combining items (i) and (ii), which implies lower expected payoff for the firm as well.

Any alternative reporting system can be obtained via combinations of the adjustments described in items (i), (ii), and (iii), each of which yields lower payoff utility for the firm. Therefore, $\boldsymbol{\beta}^*$ is the reporting system that maximizes the firm's expected payoff.

Proof of Corollary 1: When $k > \widehat{k}$, the reporting cutoff is chosen as described in the proof of Proposition 1. We can exploit the implicit function theorem on $G(\widehat{\omega}) = 0$ as defined in (A.7) to calculate the derivatives. We have,

$$\frac{d\widehat{\omega}}{dp} = \frac{-\frac{\partial G(\widehat{\omega})}{\partial p}}{\frac{\partial G(\widehat{\omega})}{\partial \widehat{\omega}}} = \frac{1-F(\widehat{\omega})+\frac{p}{1-p}\left(F(k)-F(\widehat{\omega})\right)}{\frac{f(\widehat{\omega})(k-\widehat{\omega})}{1-p}} > 0, \text{ and} \tag{A.10}$$

$$\frac{d\widehat{\omega}}{dk} = \frac{-\frac{\partial G(\widehat{\omega})}{\partial k}}{\frac{\partial G(\widehat{\omega})}{\partial \widehat{\omega}}} = \frac{\frac{1}{(1-p)^2}\left(\int_{\widehat{\omega}}^{k} F(\omega)d\omega - F(\widehat{\omega})(k-\widehat{\omega})\right)}{\frac{f(\widehat{\omega})(k-\widehat{\omega})}{1-p}} > 0. \tag{A.11}$$

Proof of Corollary 2: The proof follows directly from applying Corollary 1 to the firm's expected payoff in Proposition 1, part (ii).

Proof of Corollary 3: The proof exploits the result from Shaked and Shanthikumar (2007) that if $F_1(\omega)$ hazard ratio dominates $F_2(\omega)$, then for any x, the distribution F_1 conditional on $\omega > x$ first-order stochastic dominates the distribution of F_2 conditional on $\omega > x$, i.e., $F_2(\omega|\omega > x) \leq_{st} F_1(\omega|\omega > x)$, $\forall t$, where $A \leq_{st} B$ denotes B first-order stochastic dominates A. Let $x = \widehat{\omega}_1$ and $t_{g,i}$ be the disclosure cutoff following a good report under prior distribution $F_i(\omega)$, $i \in \{1, 2\}$. Proposition 3 from Jung and Kwon (1988) implies that $t_{g,2} < t_{g,1} = k$. Note that k has not changed, so the firm will optimally shift $\widehat{\omega}_2$ up to restore $t_{g,2} = k$. Therefore, if $F_2(\omega)$ hazard ratio dominates $F_1(\omega)$, then $\widehat{\omega}_2 \geq \widehat{\omega}_1$.

Proof of Corollary 4: The proof follows directly from the fact that Pr(disclosure) $= p\left(1-F(k)\right)$.

Proof of Corollary 5: The probability of disclosure with a binary reporting system is

$$\begin{aligned}\Pr\left(\text{disclosure}\right) &= \Pr\left(\text{disclosure}|r_g\right)\Pr\left(r_g\right)+\Pr\left(\text{disclosure}|r_b\right)\Pr\left(r_b\right)\\ &= p\left(1-F\left(k\right)\right)+p\left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right)\end{aligned}$$

Total derivatives of Pr(*disclosure*) *w.r.t. k and p*: Clearly, the first portion of the expression for Pr(disclosure) above, $\Pr(\text{disclosure})\Pr\left(r_g\right)=p\left(1-F\left(k\right)\right)$, is increasing in p and decreasing in k, as for an impairment-style reporting system. For the second portion, we have

$$\begin{aligned}\frac{d\Pr\left(\text{disclosure}\cap r_g\right)}{dp} &= \left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right)+p\left(f\left(\widehat{\omega}\right)\frac{d\widehat{\omega}}{dp}-f\left(t_b\right)\frac{dt_b}{dp}\right)\\ &= \left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right)+p\left(f\left(\widehat{\omega}\right)\frac{d\widehat{\omega}}{dp}-f\left(t_b\right)\left(\frac{\partial t_b}{\partial p}+\frac{\partial t_b}{d\widehat{\omega}}\frac{d\widehat{\omega}}{dp}\right)\right)\\ &= \left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right)+p\left(\frac{d\widehat{\omega}}{dp}\left(f\left(\widehat{\omega}\right)-f\left(t_b\right)\frac{\partial t_b}{\partial\widehat{\omega}}\right)-f\left(t_b\right)\frac{\partial t_b}{\partial p}\right).\end{aligned}$$

Note that $\frac{d\widehat{\omega}}{dp}=\frac{k\left(F(k)-F\left(\widehat{\omega}\right)\right)-\int_{\widehat{\omega}}^{k}\omega f(\omega)d\omega}{(1-p)f\left(\widehat{\omega}\right)\left(k-\widehat{\omega}\right)}>0$, $\frac{\partial t_b}{\partial\widehat{\omega}}=\frac{dt_b}{d\widehat{\omega}}=\frac{f\left(\widehat{\omega}\right)\left(\widehat{\omega}-t_b\right)(1-p)}{pF(t_b)+(1-p)F\left(\widehat{\omega}\right)}>0$, and $\frac{\partial t_b}{\partial p}=-\frac{F\left(\widehat{\omega}\right)\left(\widehat{\omega}-t_b\right)-\int_{t_b}^{\widehat{\omega}}F(\omega)d\omega}{pF(t_b)+(1-p)F\left(\widehat{\omega}\right)}<0$. Hence:

$$\begin{aligned}\frac{d\Pr\left(\text{disclosure}\ \cap\ r_g\right)}{dp} &= \left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right)+p\frac{d\widehat{\omega}}{dp}f\left(\widehat{\omega}\right)\left(1-\frac{f\left(t_b\right)\left(\widehat{\omega}-t_b\right)(1-p)}{pF\left(t_b\right)+(1-p)F\left(\widehat{\omega}\right)}\right)\\ &\quad +pf\left(t_b\right)\frac{F\left(\widehat{\omega}\right)\left(\widehat{\omega}-t_b\right)-\int_{t_b}^{\widehat{\omega}}F\left(\omega\right)d\omega}{pF\left(t_b\right)+(1-p)F\left(\widehat{\omega}\right)}\\ &= \left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right)+p\frac{d\widehat{\omega}}{dp}f\left(\widehat{\omega}\right)\\ &\quad +p\frac{f\left(t_b\right)\left(F\left(\widehat{\omega}\right)\left(\widehat{\omega}-t_b\right)-\int_{t_b}^{\widehat{\omega}}F\left(\omega\right)d\omega-\frac{d\widehat{\omega}}{dp}f\left(\widehat{\omega}\right)\left(\widehat{\omega}-t_b\right)(1-p)\right)}{pF\left(t_b\right)+(1-p)F\left(\widehat{\omega}\right)}\\ &= \left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right)+p\frac{d\widehat{\omega}}{dp}f\left(\widehat{\omega}\right)\\ &\quad +p\frac{f\left(t_b\right)\left(\begin{array}{c}\left(\widehat{\omega}-t_b\right)\left(F\left(\widehat{\omega}\right)\left(k-\widehat{\omega}\right)+F\left(k\right)k-F\left(\widehat{\omega}\right)\widehat{\omega}\right)\\ -\left(k-\widehat{\omega}\right)\int_{t_b}^{\widehat{\omega}}F\left(\omega\right)d\omega-\left(\widehat{\omega}-t_b\right)\int_{\widehat{\omega}}^{k}F\left(\omega\right)d\omega\end{array}\right)}{\left(k-\widehat{\omega}\right)\left(pF\left(t_b\right)+(1-p)F\left(\widehat{\omega}\right)\right)}.\end{aligned}$$

Simplifying,

$$\begin{aligned}
\frac{d\Pr\left(\text{disclosure } \cap \ r_g\right)}{dp} &= \left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right)+p\frac{d\widehat{\omega}}{dp}f\left(\widehat{\omega}\right)+p\frac{f\left(t_b\right)\widehat{\omega}\left(\widehat{\omega}-t_b\right)\left(F\left(k\right)-F\left(\widehat{\omega}\right)\right)}{\left(k-\widehat{\omega}\right)\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)\right)} \\
&\quad +p\frac{f\left(t_b\right)\left(\left(k-\widehat{\omega}\right)\left(F\left(\widehat{\omega}\right)\left(\widehat{\omega}-t_b\right)-\int_{t_b}^{\widehat{\omega}}F\left(\omega\right)d\omega\right)\right)}{\left(k-\widehat{\omega}\right)\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)\right)} \\
&\quad +p\frac{f\left(t_b\right)\left(\left(\widehat{\omega}-t_b\right)\left(F\left(k\right)\left(k-\widehat{\omega}\right)-\int_{\widehat{\omega}}^{k}F\left(\omega\right)d\omega\right)\right)}{\left(k-\widehat{\omega}\right)\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)\right)} \geq 0,
\end{aligned}$$

where the inequality follows from F increasing, $k \geq \widehat{\omega} \geq t_b$, and $F\left(y\right)\left(y-x\right)-\int_x^y F\left(\omega\right)d\omega \geq 0 \forall y \geq x$ in the domain of F.

For changes in the probability of disclosure following changes in k, we can take total derivatives on the probability of disclosure:

$$\begin{aligned}
\Pr(\text{disclosure}) &= \Pr(\text{disclosure})\Pr\left(r_g\right)+\Pr\left(\text{disclosure}|r_b\right)\Pr\left(r_b\right) \\
&= p\left(1-F\left(k\right)\right)+p\left(F\left(\widehat{\omega}\right)-F\left(t_b\right)\right).
\end{aligned}$$

Using (A.10) and (A.11), and integration by parts for $\frac{d\widehat{\omega}}{dp}$ and $\frac{dt_b}{d\widehat{\omega}}=\frac{f\left(\widehat{\omega}\right)\left(\widehat{\omega}-t_b\right)\left(1-p\right)}{pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)}>0$, the total derivative with respect to k is:

$$\begin{aligned}
\frac{d}{dk}\Pr(\text{disclosure}) &= -pf\left(k\right)+p\frac{d\widehat{\omega}}{dk}\left(f\left(\widehat{\omega}\right)-f\left(t_b\right)\frac{dt_b}{d\widehat{\omega}}\right) \\
&= -pf\left(k\right) \\
&\quad +p\frac{\left(1-p\right)\left(1-F\left(\widehat{\omega}\right)\right)+p\left(F\left(k\right)-F\left(\widehat{\omega}\right)\right)}{f\left(\widehat{\omega}\right)\left(k-\widehat{\omega}\right)} \\
&\quad \times\left(f\left(\widehat{\omega}\right)-f\left(t_b\right)\frac{f\left(\widehat{\omega}\right)\left(\widehat{\omega}-t_b\right)\left(1-p\right)}{pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)}\right) \\
&= -pf\left(k\right) \\
&\quad +p\frac{\left(1-p\right)\left(1-F\left(\widehat{\omega}\right)\right)+p\left(F\left(k\right)-F\left(\widehat{\omega}\right)\right)}{\left(k-\widehat{\omega}\right)}\left(1-\frac{f\left(t_b\right)\left(\widehat{\omega}-t_b\right)\left(1-p\right)}{pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)}\right) \\
&= p\left(\left(1-p\right)\left(1-F\left(\widehat{\omega}\right)\right)+p\left(F\left(k\right)-F\left(\widehat{\omega}\right)\right)\right) \\
&\quad \times\frac{\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)-f\left(t_b\right)\left(\widehat{\omega}-t_b\right)\left(1-p\right)\right)}{\left(k-\widehat{\omega}\right)\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)\right)} \\
&\quad -\frac{pf\left(k\right)\left(k-\widehat{\omega}\right)\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)\right)}{\left(k-\widehat{\omega}\right)\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)\right)} \\
&\propto -\left(\left(1-p\right)\left(1-F\left(\widehat{\omega}\right)\right)+p\left(F\left(k\right)-F\left(\widehat{\omega}\right)\right)\right)f\left(t_b\right)\left(\widehat{\omega}-t_b\right)\left(1-p\right) \\
&\quad +\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)\right)\left(\left(1-p\right)\left(1-F\left(\widehat{\omega}\right)\right)+p\left(F\left(k\right)-F\left(\widehat{\omega}\right)\right)\right) \\
&\quad -\left(pF\left(t_b\right)+\left(1-p\right)F\left(\widehat{\omega}\right)\right)f\left(k\right)\left(k-\widehat{\omega}\right).
\end{aligned}$$

From numerical examples (provided in the main text), this can be either positive or negative.

Proof of Proposition 2: Note that

$$\begin{aligned} E\left[\pi|Discretionary\ Disclosure\right] &> E\left[\pi|Mandatory\ Disclosure\right] \\ \Leftrightarrow E\left[\omega\right]-CF\left(\widehat{\omega}\right) &> E\left[\omega\right]-C\left(pF\left(k\right)+\left(1-p\right)F\left(\widehat{\omega}^{0}\right)\right) \\ \Leftrightarrow F\left(\widehat{\omega}\right) &< pF\left(k\right)+\left(1-p\right)F\left(\widehat{\omega}^{0}\right) \\ \Leftrightarrow F\left(\widehat{\omega}\right)-F\left(\widehat{\omega}^{0}\right) &< p\left(F\left(k\right)-F\left(\widehat{\omega}^{0}\right)\right). \end{aligned}$$

Define

$$H\left(p\right)=p\left(F\left(k\right)-F\left(\widehat{\omega}^{0}\right)\right)-\left(F\left(\widehat{\omega}\right)-F\left(\widehat{\omega}^{0}\right)\right).$$

If this is positive on $p\in[0,1]$, then the firm will always prefer discretionary disclosure. Note that $H\left(p\right)$ is zero at the boundaries, as the limits are

$$\begin{aligned} \lim_{p\to 0}H\left(p\right) &= 0\left(F\left(k\right)-F\left(\widehat{\omega}^{0}\right)\right)-\left(F\left(\widehat{\omega}^{0}\right)-F\left(\widehat{\omega}^{0}\right)\right)=0,\text{ and} \\ \lim_{p\to 1}H\left(p\right) &= \left(F\left(k\right)-F\left(\widehat{\omega}^{0}\right)\right)-\left(F\left(k\right)-F\left(\widehat{\omega}^{0}\right)\right)=0. \end{aligned}$$

Note that $\lim_{p\to 1}\widehat{\omega}=k$ follows from the following condition holding as $p\to 1$:

$$\left(1-p\right)\left(E\left[\omega|r_g\right]-k\right)=p\int_{\widehat{\omega}}^{k}F_g\left(\omega\right)d\omega.$$

Using (A.10), the first derivative of $H\left(p\right)$ is

$$\begin{aligned} \frac{dH\left(p\right)}{dp} &= F\left(k\right)-F\left(\widehat{\omega}^{0}\right)-f\left(\widehat{\omega}\right)\frac{d\widehat{\omega}}{dp} \\ &= F\left(k\right)-F\left(\widehat{\omega}^{0}\right)-f\left(\widehat{\omega}\right)\frac{\int_{\widehat{\omega}}^{k}F\left(\omega\right)d\omega-F\left(\widehat{\omega}\right)\left(k-\widehat{\omega}\right)}{f\left(\widehat{\omega}\right)\left(k-\widehat{\omega}\right)\left(1-p\right)} \\ &= F\left(k\right)-F\left(\widehat{\omega}^{0}\right)-\frac{\int_{\widehat{\omega}}^{k}F\left(\omega\right)d\omega}{\left(k-\widehat{\omega}\right)\left(1-p\right)}+\frac{F\left(\widehat{\omega}\right)}{\left(1-p\right)}. \end{aligned}$$

Recall that $\lim_{p\to 0}\widehat{\omega}=\widehat{\omega}^{0}$ and $\lim_{p\to 1}\widehat{\omega}=k$. At the lower-bound of $p=0$, we have

$$\begin{aligned} \lim_{p\to 0}\frac{dH\left(p\right)}{dp} &= F\left(k\right)-F\left(\widehat{\omega}^{0}\right)+F\left(\widehat{\omega}^{0}\right)-\frac{\int_{\widehat{\omega}^{0}}^{k}F\left(\omega\right)d\omega}{\left(k-\widehat{\omega}^{0}\right)} \\ &= F\left(k\right)-\frac{\int_{\widehat{\omega}^{0}}^{k}F\left(\omega\right)d\omega}{\left(k-\widehat{\omega}^{0}\right)} \\ &\geq F\left(k\right)-\frac{F\left(k\right)\left(k-\widehat{\omega}^{0}\right)}{\left(k-\widehat{\omega}^{0}\right)}=0, \end{aligned}$$

where the inequality follows from $\int_{\widehat{\omega}^{0}}^{k}F\left(\omega\right)d\omega\leq F\left(k\right)\left(k-\widehat{\omega}^{0}\right)$, which is a result of the area under $F\left(\omega\right)$ from $\widehat{\omega}^{0}$ to k being less than the area under the rectangle of height $F\left(k\right)$ and

width $k-\widehat{\omega}^0$. At the upper-bound of $p=1$, we have

$$\begin{aligned}
\lim_{p\to 1}\frac{dH(p)}{dp} &= F(k)-F\left(\widehat{\omega}^0\right)-\frac{\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega}{(k-k)(1-p)}+\frac{F(k)}{(1-p)}\\
&= F(k)-\frac{\int_{\widehat{\omega}^0}^{k}F(\omega)\,d\omega}{(k-\widehat{\omega}^0)}\\
&\geq F(k)-\frac{F(k)(k-\widehat{\omega}^0)}{(k-\widehat{\omega}^0)}=0,
\end{aligned}$$

where the inequality follows from $\int_{\widehat{\omega}^0}^{k}F(\omega)\,d\omega\leq F(k)(k-\widehat{\omega}^0)$, which is a result of the area under $F(\omega)$ from $\widehat{\omega}^0$ to k being less than the area under the rectangle of height $F(k)$ and width $k-\widehat{\omega}^0$. The second derivative of $H(p)$ is:

$$\begin{aligned}
\frac{d^2H(p)}{dp^2} &= -\frac{d}{dp}\frac{\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})}{(k-\widehat{\omega})(1-p)}\\
&= -\frac{\frac{d}{dp}\left(\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})\right)(k-\widehat{\omega})(1-p)}{\left((k-\widehat{\omega})(1-p)\right)^2}\\
&\quad+\frac{\left(\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})\right)\frac{d}{dp}(k-\widehat{\omega}+p\widehat{\omega}-kp)}{\left((k-\widehat{\omega})(1-p)\right)^2}.
\end{aligned}$$

Simplifying,

$$\begin{aligned}
\frac{d^2H(p)}{dp^2} &\propto -\left(-F(\widehat{\omega})\frac{d\widehat{\omega}}{dp}-f(\widehat{\omega})\frac{d\widehat{\omega}}{dp}(k-\widehat{\omega})+F(\widehat{\omega})\frac{d\widehat{\omega}}{dp}\right)(k-\widehat{\omega})(1-p)\\
&\quad-\left(\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})\right)\left(\frac{d\widehat{\omega}}{dp}(1-p)+k-\widehat{\omega}\right)\\
&= f(\widehat{\omega})\frac{d\widehat{\omega}}{dp}(k-\widehat{\omega})^2(1-p)\\
&\quad-\left(\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})\right)\left(\frac{d\widehat{\omega}}{dp}(1-p)+k-\widehat{\omega}\right)\\
&= f(\widehat{\omega})\frac{\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})}{f(\widehat{\omega})(k-\widehat{\omega})(1-p)}(k-\widehat{\omega})^2(1-p)\\
&\quad-\left(\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})\right)\\
&\quad\times\left(\frac{\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})}{f(\widehat{\omega})(k-\widehat{\omega})(1-p)}(1-p)+k-\widehat{\omega}\right)\\
&= -\frac{\left(\int_{\widehat{\omega}}^{k}F(\omega)\,d\omega-F(\widehat{\omega})(k-\widehat{\omega})\right)^2}{f(\widehat{\omega})(k-\widehat{\omega})}<0.
\end{aligned}$$

The second derivative of $H(p)$ is negative, so the function is concave. Furthermore, it is 0 and increasing at $p=0$, and crosses 0 again at $p=1$. This implies that it is greater than 0

on $(0,1)$.

Proof of Proposition 4: We introduce notation from Kamenica and Gentzkow (2011). Let $\widehat{\pi}^o\left(F_{r_j,\boldsymbol{\beta}}\right) \equiv E_m[\pi^o(r_j, m, \boldsymbol{\beta})]$ be the expected payoff over any possible messages, conditional on beliefs $F_{r_j,\boldsymbol{\beta}}$, and let $\widehat{\Pi}^o\left(F_{r_j,\boldsymbol{\beta}}\right)$ be the concavification of $\widehat{\pi}^o\left(F_{r_j,\boldsymbol{\beta}}\right)$. From Kamenica and Gentzkow (2011), we know that the optimal reporting system is given by reports that induce beliefs at points of intersection between $\widehat{\pi}^o$ and $\widehat{\Pi}^o$.

For high k, flattening the portion of the objective below the threshold k can lead to two possibilities. First, it may be that $\widehat{\pi}^o$ and $\widehat{\Pi}^o$ intersect at beliefs that imply $E[\omega|r, m = \varnothing, \boldsymbol{\beta}] = k$, in which case the solution will be equivalent to that described in the main part of the paper. Let $\boldsymbol{\beta}^*$ be an optimal reporting system defined by Proposition 1. The firm's expected payoff in this case will be

$$E\left[\pi^o(r, m, \boldsymbol{\beta}^*)\right] = E[\omega] - F(\widehat{\omega})\left(C + \delta E[\omega|\omega < \widehat{\omega}]\right). \tag{A.12}$$

Second, it may be that the flattening causes $\widehat{\Pi}^o$ to be above $\widehat{\pi}^o$ at the beliefs that imply $E[\omega|r, m = \varnothing, \boldsymbol{\beta}] = k$, so that $\boldsymbol{\beta}^*$ is no longer optimal. Instead, the concavification will be as with a globally convex objective. In this case, an optimal reporting system is perfectly informative (Kamenica and Gentzkow 2011, p. 2602). Let $\boldsymbol{\beta}^P$ be a perfectly informative reporting system (i.e., with $r_i = \omega_i \ \forall \omega_i \in \Omega$). The firm's expected payoff with $\boldsymbol{\beta}^P$ is

$$E\left[\pi^o\left(r, m, \boldsymbol{\beta}^P\right)\right] = E[\omega] - F(k)\left(C + \delta E[\omega|\omega < k]\right). \tag{A.13}$$

Comparing equations (A.12) and (A.13) implies that the firm will prefer an imperfectly informative reporting system defined by $\boldsymbol{\beta}^*$ when $C > C^o$, where

$$C^o \equiv \delta \frac{F(\widehat{\omega})E[\omega|\omega < \widehat{\omega}] - F(k)E[\omega|\omega < k]}{F(k) - F(\widehat{\omega})}. \tag{A.14}$$

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