

Investments and Risk Transfers*

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Abstract

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We demonstrate a novel link between relationship-specific investments and risk in a setting where division managers operate under moral hazard and collaborate on joint projects. Specific investments increase efficiency at the margin. This expands the scale of operations and thereby adds to the compensation risk borne by the managers. Accounting for this investment/risk link overturns key findings from prior incomplete contracting studies. We find that, if the investing manager has full bargaining power vis-a-vis the other manager, he will underinvest relative to the benchmark of contractible investments; with equal bargaining power, however, he may overinvest. The reason is that the investing manager internalizes only his own share of the investment-induced risk premium (we label this a “risk transfer”), whereas the principal internalizes both managers’ incremental risk premia. We show that high pay-performance sensitivity (PPS) reduces the managers’ incentives to invest in relationship-specific assets. The optimal PPS thus trades off investment and effort incentives.

Keywords: incomplete contracting, hold-up, risk externalities, overinvestment, pay performance sensitivity (PPS)

1 Introduction

The joint provision of effort or financial resources is a central feature of modern firms. Yet, such arrangements are prone to externality problems such as free-riding or hold-up (Holmstrom 1982, Williamson 1985). In fact, a key tenet of the incomplete contracting literature is that individual agents underinvest in joint projects unless they expect to fully extract the cash returns *ex post*, i.e., have complete bargaining power.¹ The issue of risk associated with joint projects is typically not addressed. Yet, underlying the incomplete contracting framework is the notion of state uncertainty, which one would expect to translate into compensation risk. We show that formally modeling the risk consequences of relationship-specific investments overturns several key results from earlier studies. For instance, a division manager who has full bargaining power vis-a-vis his counterpart always underinvests, whereas, paradoxically, he may *overinvest* if he expects to split the surplus.

The driving force behind these findings is that efficiency-enhancing investments in joint projects increase managers' compensation risk.² Consider the canonical surplus splitting model in which divisions A and B trade widgets made by A at unit cost of $\$c$ and sold by B to external customers, where c is a random variable realized just before production. At the outset, A can install a more efficient machine at some fixed cost that shifts the distribution over c to the left and thereby raises the ex-post efficient trading volume, pointwise. Ex ante, however, each unit to be traded is subject to the cost shock embedded in c . Hence, greater upfront investment not only increases the expected value but also the variance of the surplus, which in turn tends to translate into additional variance in man-

¹See Williamson (1985). A stream of the incomplete contracting literature has studied ways of augmenting the bargaining process to overcome the hold-up problem; e.g., Chung (1991), Rogerson (1992), Edlin and Reichelstein (1995).

²We only consider investments in assets that enhance the efficiency of operations, and thereby add to the scale of the latter and ultimately, as we show, to overall risk. Other investments, such as hedging, are specifically designed to reduce risk. We ignore those here.

agers' compensation.³ Moreover, a manager who expects to split the surplus externalizes not only a share of the cash returns (the standard hold-up problem), muting his investment incentives, but also a share of the incremental risk premium, boosting his investment incentives. We label this effect a *risk transfer*. It raises the possibility of overinvestment under incomplete contracts.

We study the investment/risk link and its implications for contract design and investment distortions in a setting in which a principal contracts with two divisional managers who collaborate on a joint project. The value of the project can be enhanced by an upfront specific investment. We first consider general (nonlinear) but exogenous compensation schemes. In the *benchmark setting of contractible investments*, when choosing the investment, the principal internalizes all cash flows and reimburses both managers for their risk premia. In contrast, a manager who has been delegated investment authority and who has full bargaining power ex-post vis-a-vis his counterpart internalizes the entire incremental risk premium *directly*, whereas cash flows—the investment-related fixed cost and contribution margin—accrue in his divisional income measure and thus *flow through the compensation scheme*. Unless the manager at the margin pockets each dollar of divisional income as compensation (i.e., he is residual claimant at the margin vis-a-vis the other manager *and* vis-a-vis the principal), he will overweight the risk premium and hence underinvest.

As bargaining power becomes more evenly distributed, one would expect the hold-up effect to compound the underinvestment problem. Yet, the risk transfer effect constitutes a countervailing force. To study the ensuing tradeoff, we turn to linear contracts and equal-split bargaining between the managers, which also allows us to derive the optimal pay-performance sensitivity (PPS). We

³As we show in Lemma 1, this argument holds for linear, and a fortiori also for convex compensation schemes. For concave contracts, investments that add to the first and second moments of the outcome distribution have an ambiguous effect on the risk premium: the outcome distribution, while more dispersed, now falls into a flatter region of the contract.

derive sufficient conditions for either of the countervailing externalities—hold-up or risk transfer—to dominate, thereby predicting the direction of the investment distortion. Underinvestment results if the project uncertainty is small or the non-investing manager faces volatile general operations, muting his PPS. Both forces dampen the risk transfer effect. The earlier hold-up studies that have ignored project-related compensation risk thus emerge as a limit case of our model.

On the other hand, overinvestment results if the benefits from the joint project are highly uncertain and the investing manager faces a more volatile operating environment than his non-investing counterpart. Together these two forces magnify the risk transfer effect: the project risk is high, and the non-investing manager internalizes a large share of the associated risk premium because of his relatively high PPS. The possibility of overinvestment illustrates the importance of recognizing the investment/risk link.

The investment/risk link has implications for contract design. Managers operating under high-powered incentives are reluctant to invest, all else equal, because they are more sensitive to the incremental variance in divisional income. Therefore, managers facing similar divisional agency problems but divergent investment opportunities should receive different incentive contracts, because the PPS now serves as an instrument to fine-tune delegated investment decisions.

A principal primarily concerned with a prevailing underinvestment problem (due to hold-up) can lower the investing manager's PPS to stimulate investment. However, the PPS reduction required to induce the benchmark investment level would impose too high an opportunity cost in terms of foregone effort; hence, some degree of underinvestment remains in equilibrium. As a consequence, the non-investing manager's PPS exceeds that in the contractible benchmark setting because less investment reduces the marginal risk premium. For the investing manager, this risk effect has to be traded against the investment-inducing effect of low PPS. If the operating volatility faced by the investing manager is sufficiently

high, the investment inducement effect dominates, resulting in muted incentives. Converse arguments apply if risk transfer is the dominant friction, resulting in overinvestment.

In sum, accounting for the risk consequences of value-enhancing investments overturns several earlier results. A manager with full bargaining power vis-a-vis his counterpart underinvests in specific assets; yet, equal-split bargaining may result in overinvestment. Hence, investment distortions arising from incomplete contracting can be non-monotonic in the allocation of bargaining power. Moreover, the optimal PPS balances effort and investment incentives as high-powered incentives tend to suppress investments. Thus, the equilibrium association between distortions in investments and PPS is always negative for managers without investment opportunities, but it may take either sign for managers with investment opportunities. That is, investment opportunities at the divisional level are an omitted correlated variable when relating risk measures to PPS.

Earlier papers studying specific investments jointly with divisional moral hazard problems are Holmstrom and Tirole (1991) and Anctil and Dutta (1999). Both papers highlight the investment-enhancing role of profit sharing. Absent profit sharing, they predict separation of PPS and investment incentives in that (a) delegated investments are independent of the PPS and, conversely, (b) managers facing similar divisional agency problems receive the same PPS regardless of their respective investment opportunities, because the investment is paid for with divisional funds and is not personally costly. This implicit dichotomy of “vertical” agency problems between principals and managers—addressed by the PPS—and “horizontal” coordination among managers—addressed by transfer pricing mechanisms—assumes away the investment/risk link. By showing that investments are decreasing in the PPS, our results establish a link between incentive pay and investments, even absent profit sharing. This illustrates the importance of modeling residual cash flow claims comprehensively by incorpo-

rating principal-agent incentive contracting and inter-agent surplus-splitting.

Williamson (1985) has observed that incentives in vertically integrated firms are typically low-powered, in part to prevent managers from misallocating their efforts across various tasks. Holmstrom and Milgrom (1991) and Feltham and Xie (1994) have formalized this argument using multitasking models in which effort that is easily measurable, but not necessarily productive, crowds out other dimensions of performance that are essential but hard to measure.⁴ In our model, each task, effort and investment, in general has a positive impact on firm value and on divisional performance measures. Yet, muting incentives below the level predicted by standard moral hazard models may be optimal because high-powered PPS depresses managers' investment incentives.

Several papers have augmented the moral hazard model by allowing for actions taken by an agent to affect the variance of the outcome.⁵ We demonstrate that a second moment-effect arises endogenously even from actions that have traditionally been viewed as affecting only the mean of the outcome distribution, namely investments in efficiency-enhancing fixed assets. Lastly, our findings linking managers' risk-taking incentives to the curvature of their compensation contracts are related to earlier finance studies on managers' risk-taking incentives under option contracts.⁶

The article proceeds as follows. Section 2 develops the investment/risk link in a general setup. Section 3 adds more structure to the model to facilitate a

⁴See also Christensen, Sabac and Tian (2010), and Heinle, Hofmann, and Kunz (2012).

⁵For instance, Prendergast (2002), Baker and Jorgensen (2003), Bertomeu (2008), and Liang and Nan (2014) have studied single-agent models with effort affecting the first and second moments of output. Ziv (2000), Liang et al. (2008), Indjejikian and Matejka (2009) and Friedman (2013, 2014) are closer to our model in that they have considered multi-agent models. What sets our model apart: (a) we consider a joint production technology; (b) the second-moment effect in our model arises endogenously from efficiency-enhancing investments that scale up the operations in line with the incomplete contracting literature. In contrast, in the earlier papers, some effort taken by an agent *directly* affects the outcome variance.

⁶Carpenter (2000) and Ross (2004) have shown that stock options may induce less risk-taking if risky projects move the outcome distribution towards the domain region with greater contract slope. Armstrong et al. (2013) conduct empirical analysis along these lines.

detailed analysis of the optional (linear) contracts. Section 4 addresses the benchmark model with contractible investments. Section 5 deals with non-contractible investments. Section 6 concludes. All proofs are found in the Appendix unless otherwise noted.

2 The Investment/Risk Link

In this section, we present a reduced-form version of the full-fledged model. For now, we take the managers' compensation contracts as given and suppress divisional agency problems so as to focus on the joint project and develop the intuition for the fundamental link between specific investments and risk.

A principal contracts with two division managers, $i = A, B$. Each Manager i is compensated according to some exogenous function $s_i(\pi_i)$ based on own-division performance, π_i . We assume that the contracts are strictly increasing, i.e., $s'_i(\cdot) > 0$, for any i, π_i , for reasons to be made explicit in Sections 3-5. There, we also comment on the issues of divisional performance evaluation and firm-wide profit sharing.

The managers collaborate on a joint project that generates surplus $M(\theta, I)$, which is increasing in some upfront investment, $I \geq 0$, and in a random state variable, θ , i.e., $M_I > 0$, $M_\theta > 0$. Moreover, we impose the single-crossing condition $M_{\theta I} > 0$, which is a standard feature of hold-up models.⁷ To fix ideas, suppose the investment is efficiency-enhancing at the margin, e.g., it may lower the variable cost per unit of the joint project, while θ is a (favorable) random shock to the size of the market for the project. The investment I is undertaken by Manager A at fixed cost $F(I)$ for Division A. We consider cases where I is or is not contractible.

After the investment is made, the managers jointly observe the realization of

⁷See, e.g., the value function in Pfeiffer et al. (2011) in a transfer pricing setting.

θ but cannot communicate it to the principal.⁸ The managers bargain over the surplus with the result that Manager i receives a share $\gamma_i M$, with $\gamma_A \in [0, 1]$ and $\gamma_B = 1 - \gamma_A$. Realized divisional profits are

$$\pi_i(\theta, I) = \gamma_i M(\theta, I) - \mathbb{1}_{i=A} F(I), \quad (1)$$

with $\mathbb{1}_{i=A}$ as the indicator function. Figure 1 depicts the sequence of events.

— INSERT FIGURE 1 HERE —

We assume throughout that the principal is risk-neutral and the managers are risk-averse with mean-variance preferences:

$$EU_i(I \mid \gamma_i) = E[s_i(\pi_i(\theta, I))] - \frac{\rho_i}{2} Var(s_i(\pi_i(\theta, I))), \quad (2)$$

where $\rho_i \geq 0$ is Manager i 's coefficient of risk aversion. Absent pre-contractual private information or other frictions, the managers will earn zero rents, i.e., at Date 1 the principal extracts the surplus net of the managers' risk premia and realizes an expected payoff of:

$$\Pi(I) = E[M(\theta, I)] - F(I) - \sum_{i=A,B} \frac{\rho_i}{2} Var(s_i(\pi_i(\theta, I))). \quad (3)$$

If the investment is contractible, the principal chooses I so as to maximize $\Pi(I)$, for given $s_i(\cdot)$, $i = A, B$. We assume existence of a unique interior solution I^* , termed the *benchmark* solution.

Earlier treatments of specific investment have centered on the first-moment effect of investment on ex-post surplus, and on whether the investing party can capture a sufficiently high share of the surplus. As we show now, there is also a second-moment effect. With M_θ and $M_{\theta I}$ both positive, investment adds to

⁸Blocked (or imperfect) communication of realized information is a standard assumption in the incomplete contracting literature, e.g., Melumad et al. (1992) and Prendergast (2002).

the *variance in the surplus*, i.e., $Var(M(\theta, I))$ is increasing in I .⁹ How does the incremental surplus variance translate into *compensation variance*? Building on the “concavification/convexification” arguments in Carpenter (2000) and Ross (2004), the answer depends on the shape of the compensation contracts and on whether investment shifts the distribution over divisional performance measures to the left or the right.¹⁰ Given $\partial\pi_i(\theta, I)/\partial I = \gamma_i M_I(\theta, I) - \mathbb{1}_{i=A} F'(I)$, for any i and θ , we find:

Lemma 1 *For any given compensation contracts, $s_i(\cdot)$ and any $i = A, B$, the variance of compensation, $Var(s_i(\pi_i(\cdot)))$, is increasing in I at some arbitrary value $I = x$, if any of the following conditions are met:*

- (i) $s_i''(\cdot) > 0$, for any π_i , and $\left. \frac{\partial\pi_i(\theta, I)}{\partial I} \right|_{I=x} \geq 0$, for any θ ; or
- (ii) $s_i''(\cdot) < 0$, for any π_i , and $\left. \frac{\partial\pi_i(\theta, I)}{\partial I} \right|_{I=x} \leq 0$, for any θ ; or
- (iii) $|s_i''(\cdot)| < \delta$, for any π_i and for some δ positive but sufficiently small.

We illustrate Lemma 1(i), in Figure 2. The distribution over Division i 's performance measure for a high investment level, I^o , is shifted to the right and has greater dispersion, as compared with a lower investment, I_o . This translates into higher compensation risk if the contract is convex, as in panel (a). For concave contracts, panel (b), the variance effect may outweigh the first-moment effect. The distribution over $\pi_i(\cdot)$ conditional on I^o falls into a flatter region of the contract, potentially reducing the compensation risk to which Manager i is

⁹Specifically, $Var(M(\theta, I)) = E[(M(\theta, I))^2] - (E[M(\theta, I)])^2$ and hence $\partial Var(M(\cdot))/\partial I = E[2M(\cdot) \cdot M_I(\cdot)] - 2E[M(\cdot)]E[M_I(\cdot)] = 2 \cdot Cov(M(\cdot), M_I(\cdot))$. Using results in Schmidt (2003), this term is positive because $M_\theta(\cdot) > 0$ and $M_{\theta I}(\cdot) > 0$, by assumption.

¹⁰The focus in Carpenter (2000) and Ross (2004) is on how project uncertainty translates into risk premia, given nonlinear (especially, option-type) contracts and concave utility functions. Our Lemma 1 is agnostic about the shape of the managers' utility functions and instead asks how the variance in compensation is affected by upfront investments.

exposed.¹¹ Linear contracts are a special case of Lemma 1(iii): increased variance in $M(\cdot)$ then translates directly into increased variance in compensation. This observation will greatly simplify the analysis in Sections 3-5.

— INSERT FIGURE 2 HERE —

We now turn to delegated investment decisions. In many instances, divisional investments are not contractible. While *aggregate* capital expenditures are routinely monitored and verifiable, it is often difficult to trace *individual* pieces of equipment (or personnel training costs) to specific transactions. Non-contractible investments will be chosen by Manager A non-cooperatively, given his compensation contract.¹² Our goal is to compare the ensuing equilibrium investment made by Manager A with the above benchmark solution I^* . We assume that for any given contract, $s_A(\pi_A)$, there exists a unique interior optimal investment choice, denoted $I^{**}(\gamma_A)$, that solves Manager A's investment problem:

$$\max_I EU_A(I \mid \gamma_A). \quad (4)$$

We say, Manager A underinvests given γ_A , if $I^{**}(\gamma_A) < I^*$, and he overinvests if $I^{**}(\gamma_A) > I^*$.

Suppose Manager A does not share in the investment returns ($\gamma_A = 0$), but still pays for the fixed cost. With $s_A(\cdot)$ strictly increasing, Manager A would not

¹¹Reverse arguments apply to the case in which greater investment reduces a division's realized performance measure. For a concave contract, Lemma 1(ii) applies, and the manager's compensation will exhibit greater variance. Note that Lemma 1(ii) can never occur for Manager B, because this manager does not bear any fixed cost, and $M_I > 0$. Hence, Manager B free-rides on the investment, i.e., $\partial \pi_B(\theta, I)/\partial I \geq 0$ always holds. For Manager A, Lemma 1(ii), can be relevant. As will become clear below, though, our main focus is on a delegation setting in which Manager A chooses the investment level in his own interest. By revealed preference, he will never choose any I for which $\partial \pi_A(\theta, I)/\partial I < 0$.

¹²Given the equal-split bargaining protocol employed here, it is easy to see that Manager A's investment incentives depend solely on his own contract and are independent of $s_B(\cdot)$.

invest at all because of an extreme hold-up problem. On the other hand, delegating the investment decision to a manager who can make a take-it-or-leave-it offer ($\gamma_A = 1$) is generally thought to replicate the benchmark (contractible) outcome because the manager internalizes all cash flows. Recognizing the investment/risk link described in Lemma 1 proves this intuition incomplete:

Lemma 2 *If $s'_A(\pi_A) \in (0, 1)$, for any π_A , and $\gamma_A = 1$, then Manager A underinvests.*

Proof: For $\gamma_A = 1$, we have $\pi_A = M(\cdot) - F(I)$ and $\pi_B = 0$. The first-order condition for the contractible benchmark investment level, I^* (assumed unique), reads $\Pi'(I^*) = 0$, where, for any I ,

$$\Pi'(I) = E[M_I(\theta, I)] - F'(I) - \frac{\rho_A}{2} \frac{\partial \text{Var}(s_A(\pi_A(\cdot)))}{\partial I}. \quad (5)$$

If the investment is non-contractible and delegated to Manager A who faces the contract $s_A(\cdot)$ and has full bargaining power, $\gamma_A = 1$, then $I^{**}(\gamma_A = 1)$ satisfies the necessary and sufficient first-order condition $EU'_A(I^{**} \mid \gamma_A = 1) = 0$, where, for any I ,

$$\begin{aligned} EU'_A(I \mid \gamma_A = 1) &= E \left[s'_A(\pi_A(\cdot)) \cdot \frac{\partial \pi_A}{\partial I} \right] - \frac{\rho_A}{2} \frac{\partial \text{Var}(s_A(\pi_A(\cdot)))}{\partial I} \\ &= E[s'_A(\pi_A(\cdot)) \cdot (M_I(\theta, I) - F'(I))] - \frac{\rho_A}{2} \frac{\partial \text{Var}(s_A(\pi_A(\cdot)))}{\partial I}. \end{aligned} \quad (6)$$

By (5), $E[M_I(\theta, I)] - F'(I) \geq 0$ for any $I \leq I^*$. Therefore, comparing (5) and (6), by revealed preference, we have $I^* > I^{**}(\gamma_A)$, as $M_I(\theta, I) > F'(I)$, for any θ and any $I \leq I^*$, and given $s'_A(\pi_A(\cdot)) < 1$. ■

The standard intuition in prior studies was that a manager who has all bargaining power vis-a-vis the other manager invests optimally because all cash flows accrue within his performance measure. Any increasing incentive contract would constitute a monotone transformation, leaving the optimal solution unaffected.

However, this intuition ignores the investment/risk link. By Lemma 1, a change in I affects the compensation risk. Given $\gamma_A = 1$, by (5) and (6), Manager A under delegation internalizes the same risk premium at the margin as does the principal under centralization. This alignment argument does not extend to the investment-related cash flows. Ex ante, the principal fully internalizes the cash flows, whereas they “flow through compensation” for Manager A. Given $s'_i(\cdot) < 1$, Manager A undervalues cash flows relative to the risk premium.¹³ Benchmark investments would result if the manager were residual claimant, at the margin, vis-a-vis the other manager ($\gamma_A = 1$) *and* vis-a-vis the principal ($s'(\cdot) \equiv 1$). That is, a complete picture of a manager’s investment incentives requires recognizing his residual cash flow claims from both “horizontal” inter-agent bargaining and “vertical” contracting.

Because Manager A underinvests for $\gamma_A = 0$ *and* for $\gamma_A = 1$ (the novel finding), this might suggest that he will underinvest for any $\gamma_A \in [0, 1]$. However, as we show below, a manager with interior bargaining power, $\gamma_A \in (0, 1)$, may in fact *overinvest*, i.e., invest more than the principal prefers. To illustrate, consider the objective functions of the principal and of Manager A, respectively, as per (2) and (3). Starting from $\gamma_A = 1$ (as in Lemma 2), suppose bargaining power is gradually shifted toward Manager B, i.e., γ_A decreases. The hold-up problem reduces Manager A’s investment incentives. At the same time, Manager A transfers a portion of the investment-related risk premium to Manager B who now shares in the joint project’s return. As γ_A shrinks, this risk transfer effect may eventually push toward overinvestment. Evaluating the net effect of the countervailing externalities (hold-up and risk transfer) is complicated by the fact that it involves second moments of nonlinear functions of random variables. To proceed, we impose more structure on the model.

¹³Restricting $s'_i(\cdot) < 1$ is descriptive for risk sharing reasons and to avoid any temptation for the principal to destroy output.

3 A Linear-Quadratic Model of Interdivisional Collaboration

In this section we evaluate the direction of the investment distortions under incomplete contracting by endogenizing the compensation contracts. To create a role for incentive contracts, we assume each Manager i chooses personally costly general operating effort, $a_i \in \mathbb{R}_+$ (henceforth, *effort*), to increase his divisional income at a personal disutility of $V_i(a_i)$. Restating the divisional performance metrics in (1) gives

$$\pi_i(a_i, \theta, I) = a_i + \tilde{\varepsilon}_i + \gamma_i M(\theta, I) - \mathbb{1}_{i=A} F(I),$$

where $\tilde{\varepsilon}_i$ captures Division i 's *general uncertainty*. To fix ideas, we model the joint project as intrafirm trade and impose the following structure:

- (a) Linear compensation schemes: $s_i = \alpha_i + \beta_i \pi_i$, $i = A, B$, with α_i as the fixed salary and $\beta \equiv (\beta_A, \beta_B) \in \mathbb{R}_+^2$ as the vector of pay-performance sensitivities, PPS.
- (b) Linear-quadratic revenues and costs for the joint project: $M(q, \theta, I) = R(q, \theta_B) - C(q, \theta_A, I)$, with $C(q, \theta_A, I) = (c - \theta_A - I)q$ and $R(q, \theta_B) = (r - \frac{q}{2} + \theta_B)q$, where q measures the volume of intrafirm trade. Without loss of generality, let $c = r$, with r sufficiently high to ensure nonnegative costs and revenues. Moreover, $F(I) = \frac{fI^2}{2}$, with $f > 1$.
- (c) Equal-split bargaining: $\gamma_A = \gamma_B = \frac{1}{2}$.
- (d) As for general operations, the managers' effort cost functions are quadratic, $V_i(a_i) = \frac{v_i a_i^2}{2}$, $v_i > 0$, and $E[\tilde{\varepsilon}_i] = 0$ and $Var(\tilde{\varepsilon}_i) = \sigma_i^2$, for $i = A, B$.

Given the additional structure, the managers' mean-variance preferences can be restated as:

$$EU_i = \alpha_i + \beta_i E[\pi_i(\cdot)] - \frac{\rho}{2} \beta_i^2 \left(\sigma_i^2 + \frac{Var(M(\theta, I))}{4} \right) - \frac{v}{2} a_i^2. \quad (7)$$

Bargaining provides a way of sharing project risk, as reflected in the scaling of the term $Var(M(\cdot))/4$ in the risk premium (Wilson, 1968). The principal's expected payoff equals $\Pi = E[\sum_i((1 - \beta_i)\pi_i - \alpha_i)]$. To ensure contract participation, we impose the individual rationality condition:

$$EU_i \geq 0, \quad i = 1, 2. \quad (8)$$

In our setting, α_i will be chosen to make (8) bind and leave the agents with zero rents. The timeline is given in Figure 3.¹⁴

— INSERT FIGURE 3 HERE —

With equal-split bargaining, the individual cost and revenue realizations, θ_i , are immaterial; only their sum matters for the outcome. We thus collapse them into the one-dimensional random variable $\theta \equiv \theta_A + \theta_B$, with $E[\theta] = \mu$. For given θ , the optimal level of trade then is $q^*(\theta, I) = \theta + I$ and the ex-post surplus is $M(\theta, I) \equiv M(q^*(\theta, I), \theta, I) = \frac{(\theta+I)^2}{2}$. The single-crossing condition, $M_{\theta I} \geq 0$, which played a key role in Lemma 1, thus arises endogenously in this canonical setting. In expectation over θ , $E[q^*(\theta, I)] = q^*(\mu, I) = \mu + I$. For simplicity, we assume that θ follows a discrete (two-point) distribution: with equal probability it takes values $(\mu - \sqrt{S})$ or $(\mu + \sqrt{S})$, where $\sqrt{S} < \mu$ to ensure positive quantities. Then, $Var(\theta) = S$. All noise terms, θ and ε_i , $i = A, B$, are independent. To differentiate it from the divisions' general uncertainty, $\tilde{\varepsilon}_i$, we refer to S as the *project uncertainty*, pertaining to intrafirm trade. Lastly, denote the efficient investment in a hypothetical risk-free world by $\hat{I} \in \arg \max_I E[M(\theta, I)] - \frac{fI^2}{2}$, which yields $\hat{I} = \frac{\mu}{f-1}$.

¹⁴In practice, one might expect investment to precede operating efforts. Note however that the assumption that efforts and investments are chosen simultaneously is without loss of generality in Sections 3-5 where we assume linear contracts.

Throughout the article we ignore firmwide profit sharing, which would avoid the hold-up problem if used as the sole basis for compensation. Even with profit sharing the main tension in our model would remain provided, as would be the case in most settings, managers care more about their own performance than about that of the other division.^{15,16}

4 Contractible Investment

As a benchmark, suppose that the investment I is contractible, e.g., it may entail specialized equipment used in a verifiable manner to make a particular product. In this case, the principal essentially instructs Manager A as to the investment level and, in designing compensation contracts, only needs to observe the effort incentive constraints:

$$a_i(\beta_i) \in \arg \max_{a_i} EU_i(a_i, I \mid \alpha_i, \beta_i). \quad (9)$$

In our setting, the agents' effort choices are independent of the investment level for given PPS. We collapse the principal's (net) payoff from Division i 's general operations into the function

$$\Phi_i(\beta_i) \equiv a_i(\beta_i) - \frac{v}{2}(a_i(\beta_i))^2 - \frac{\rho}{2}\beta_i^2\sigma_i^2.$$

Using (9), $\Phi'_i(0) > 0 > \Phi'_i(1)$ and $\Phi''_i(\beta_i) \leq 0$ for any $\beta_i \in [0, 1]$. Let $\beta_i^{MH} \in \arg \max_{\beta_i} \Phi_i(\beta_i)$. As a benchmark, this yields the well-known PPS in a pure moral hazard model without intrafirm trade, $\beta_i^{MH} = (1 + \rho v \sigma_i^2)^{-1}$.

¹⁵Holmstrom and Tirole (1991), Anctil and Dutta (1999) study how profit sharing trades off investment incentives and risk sharing. It is easy to see that pure profit sharing is never optimal for sufficiently large general uncertainty, σ_i^2 , due to poor risk sharing.

¹⁶By focusing on inter-divisional bargaining, we also ignore more "administered" internal pricing methods. First, cost- or market-based internal pricing would require more information held at the headquarters level (Gox and Schiller, 2007). Second, internally traded goods and services are often non-commoditized in nature, creating inefficiencies also under those methods. For instance, if the external market for the intermediate good is imperfectly competitive, pricing internally at market may result in double-marginalization (Baldenius and Reichelstein 2006, Arya and Mittendorf 2010). Third, we ignore ex-ante fixed-priced contracts to be renegotiated after the realization of the state variable, as in Edlin and Reichelstein (1994).

The principal's expected payoff with contractible investments is given by

$$\Pi(I, \beta) \equiv E[M(\theta, I)] - F(I) + \sum_{i=A,B} \left[\Phi_i(\beta_i) - \frac{\rho}{8} \beta_i^2 \cdot \text{Var}(M(\theta, I)) \right]. \quad (10)$$

As argued above, the principal extracts the entire surplus ex ante and therefore solves the following optimization program (superscript “*” indicates *contractible* investments):

Program \mathcal{P}^* : $\max_{\beta, I} \Pi(I, \beta).$

Let (I^*, β^*) denote the solution to Program \mathcal{P}^* , henceforth the *benchmark*.

It is instructive to decompose the principal's optimization problem into two steps: first, derive the optimal PPS for given investment; second, solve for the optimal level of investment. Manager i 's optimal PPS conditional on I is

$$\beta_i^o(I) = \frac{1}{1 + \rho v \left(\sigma_i^2 + \frac{\text{Var}(M(\theta, I))}{4} \right)}.$$

Denote $\beta^o(I) = (\beta_A^o(I), \beta_B^o(I))$. Recognizing the project risk pushes the PPS in our setting below β_i^{MH} . The principal picks the investment level I^* that maximizes the value function,

$$\Pi^*(I) \equiv \Pi(I, \beta^o(I)).$$

Using the Envelope Theorem:¹⁷

$$\begin{aligned} \Pi^{*'}(I) &= E[M_I(\theta, I)] - F'(I) - \frac{\rho}{8} \sum_i (\beta_i^o(I))^2 \frac{\partial \text{Var}(M(\theta, I))}{\partial I} \\ &= q^*(\mu, I) - fI - \frac{\rho S}{4} \sum_i (\beta_i^o(I))^2 q^*(\mu, I). \end{aligned} \quad (11)$$

The optimal benchmark solution calls for $\Pi^{*'}(I^*) = 0$ and $\beta_i^* = \beta_i^o(I^*)$.

¹⁷In the Appendix, equation (21), we show that the project-related variance is $\text{Var}(M(\theta, I)) = [q^*(\mu, I)]^2 S$. Using $\partial q^*(\cdot)/\partial I = 1$, we have $\partial \text{Var}(M(\theta, I))/\partial I = 2q^*(\mu, I)S$.

As a maintained regularity condition, for the remainder of the article, we bound the project uncertainty from above and say that project risk is *feasible* if and only if:¹⁸

$$S \leq \min\{S_{max}, \bar{S}\}, \text{ where } S_{max} \equiv \frac{4 \cdot \min\{\sigma_A^2, \sigma_B^2\}}{(q^*(\mu, \hat{I}))^2}, \quad \bar{S} \equiv \frac{4}{\rho \sum_i (\beta_i^{MH})^2}. \quad (12)$$

The restriction that $S \leq \bar{S}$ ensures the principal will choose a strictly positive benchmark investment level, $I^* > 0$. The restriction that $S \leq S_{max}$ ensures that the project-related compensation risk for each manager is less than his respective general risk.¹⁹ It effectively bounds the benchmark PPS so that $\beta_i^* \geq \beta_i^{min} \equiv (1 + 2\rho v \sigma_i^2)^{-1}$. Let $B \equiv [\beta_A^{min}, \beta_A^{MH}] \times [\beta_B^{min}, \beta_B^{MH}]$ denote the relevant PPS range in the benchmark setting.

Lemma 1 establishing the link between investments and compensation risk directly applies to this setting with linear contracts, and the additional structure sharpens the intuition for this finding. Upfront investments in fixed assets lower the marginal cost of producing the intermediate product, raising the equilibrium volume of intrafirm trade, pointwise.²⁰ Ex ante, however, each unit traded is subject to the random shock, θ . Greater intrafirm trade volume therefore translates into greater *ex-ante* surplus uncertainty and, given linear contracts, increased compensation risk.²¹

¹⁸We had earlier restricted $\sqrt{S} < \mu$ to ensure $q^*(\cdot) > 0$. This does not effectively constrain the solution because we can always set μ sufficiently high. The parameter restrictions in (12) can be restated in terms of the primitives: $S_{max} = 4[(f-1)/\mu f]^2 \cdot \min\{\sigma_A^2, \sigma_B^2\}$ and $\bar{S} = 4[\rho \sum_{i=A,B} (1 + \rho v \sigma_i^2)^{-2}]^{-1}$

¹⁹If the risk associated with the joint project were substantially larger than that of each unit's stand-alone operations, one would expect the divisions to be merged. On a technical level, $S \leq S_{max}$ ensures that the project-related risk premium for Manager i , $\rho/8(\beta_i^o(I))^2 \cdot \text{Var}(M(\theta, I))$, is increasing in I for any $I \leq \hat{I}$, because the direct effect on the variance of the surplus dominates the indirect effect in form of a reduced PPS.

²⁰A similar effect would obtain if the investment were downstream, say, in advertising so as to shift the marginal revenue function.

²¹This contrasts with the literature that has added actions that directly affect the outcome variance, e.g., Ziv (2000), Prendergast (2002), Baker and Jorgensen (2005), Bertomeu (2008), Liang et al. (2008), Indjejikian and Matejka (2009), Friedman (2013, 2014), and Liang and Nan (2014).

Intrafirm trade and general operations in our model are technologically separable, yet contractually intertwined: the principal optimally mutes the PPS anticipating that investments in fixed assets increase the managers' compensation risk at the margin. Thus, the costs associated with specific investments entail not just capital expenditures but also incremental risk premia (for given PPS) as well as indirect opportunity costs in that the principal optimally lowers the PPS, which in turn reduces the managers' effort input. For managers that are ex-ante symmetric except for their investment opportunities, it follows that they will face identical PPS in the benchmark setting, i.e., $\beta_A^* = \beta_B^*$ if and only if $\sigma_A^2 = \sigma_B^2$.²²

5 Non-Contractible Investment

How does contractual incompleteness, in conjunction with ex-post surplus sharing among the managers, affect investment levels and contracts? The answers given in earlier studies (e.g., Anciaux and Dutta, 1999) are: (a) incomplete contracting yields underinvestment; (b) absent profit sharing, managers' investment incentives are independent of their PPS and, conversely, the managers' PPS should be independent of their investment opportunities. As we show now, all these findings can be overturned by accounting for joint project risk.

Suppose investment I is chosen by Manager A in his own best interest. In keeping with the bulk of the incomplete contracting literature, we assume I is observable to Manager B but cannot be verified to the principal. Given the structure imposed, Manager A's effort and investment choices are mutually independent for given PPS, so we can separate out the parts of Manager A's expected

²²While the managers will receive the same PPS under the contractible benchmark solution ($\beta_A^* = \beta_B^* = \beta^*$), their fixed salaries, α_i^* , will differ. Specifically, $\alpha_A^* = \alpha_B^* + \beta^* F(I^*)$, to compensate Manager A for the fixed cost.

payoff that relate to the joint project by defining

$$\Gamma(I \mid \beta_A) \equiv \beta_A \left(\frac{E[M(\theta, I)]}{2} - F(I) - \frac{\rho}{8} \beta_A \text{Var}(M(\theta, I)) \right).$$

We denote the investment level chosen by Manager A facing a PPS of β_A by $I^{**}(\beta_A) \in \arg \max_I \Gamma(I \mid \beta_A)$. It solves the first-order condition²³

$$\Gamma'(I^{**}(\beta_A) \mid \beta_A) = 0, \quad (13)$$

where, for any I and β_A :

$$\begin{aligned} \Gamma'(I \mid \beta_A) &= \beta_A \left(\frac{E[M_I(\theta, I)]}{2} - F'(I) - \frac{\rho}{8} \beta_A \frac{\partial \text{Var}(M(\theta, I))}{\partial I} \right) \\ &= \beta_A \left(\frac{q^*(\mu, I)}{2} - fI - \frac{\rho S}{4} \beta_A q^*(\mu, I) \right). \end{aligned} \quad (14)$$

The principal's optimization program with non-contractible investments reads (superscript “**” denotes non-contractible investments):

$$\begin{aligned} \textbf{Program } \mathcal{P}^{**} : \quad & \max_{\beta, I} \Pi(I, \beta), \\ & \text{subject to (13)}. \end{aligned}$$

Denote by (I^{**}, β^{**}) the solution to this program and by $\Pi^{**}(\beta) \equiv \Pi(I^{**}(\beta_A), \beta)$ the principal's expected payoff with non-contractible investments as a function of the PPS.

A key tenet of the earlier literature is that contractual incompleteness combined with ex-post bargaining results in underinvestment. However, a comparison of the respective first-order conditions (11) and (14) shows that non-contractible investments differ from the contractible benchmark for two distinct, countervailing reasons. First, Manager A's divisional profit measure reflects only half the benefit from investing but all fixed costs—the classic hold-up problem. At the same time, the manager takes into account only his own incremental risk

²³Concavity of $\Gamma(\cdot)$ in I , for any β_A , is ensured by our maintained assumption that $f > 1$.

premium when investing and ignores that of the other manager. This *risk transfer effect* encourages *overinvestment*, all else equal. To the best of our knowledge, this effect has not been identified before.²⁴

5.1 Investment Distortions for Exogenous Contracts

To gain some preliminary understanding of the key tradeoffs, we proceed by first looking at Manager A’s investment incentives for given contracts. Specifically, we allow for any PPS within the relevant range identified above for the benchmark PPS, i.e., $\beta \in B$. Later we solve for the optimal compensation schemes under incomplete contracting.

The trade-off of the hold-up and risk transfer effects determines the direction of the investment distortion. Let $I^*(\beta) \in \arg \max_I \Pi(I, \beta)$. Adapting (11) to exogenous contracts, the principal’s marginal investment benefit for given β is

$$\frac{\partial}{\partial I} \Pi(I, \beta) = q^*(\mu, I) - fI - \frac{\rho S}{4} \sum_i \beta_i^2 q^*(\mu, I). \quad (15)$$

Differencing the principal’s and Manager A’s respective marginal investment benefits—i.e., subtracting (14) from (15)—gives:²⁵

$$\begin{aligned} \Delta(I | \beta) &\equiv \frac{\partial}{\partial I} \Pi(I, \beta) - \Gamma'(I | \beta_A) \\ &= \underbrace{q^*(\mu, I) - fI - \beta_A \left(\frac{q^*(\mu, I)}{2} - fI \right)}_{\text{hold-up effect}} - \underbrace{\frac{\rho S}{4} \beta_B^2 q^*(\mu, I)}_{\text{risk transfer effect}}. \end{aligned} \quad (16)$$

²⁴See Wilson (1968) for risk sharing in syndicates. In Wilson’s paper, the syndicate aims for a Pareto-optimal decision given the risk shared among syndicate members. In our model, the decision that affects the risk borne by the syndicate is made by one player (Manager A) in his own self interest.

²⁵In equation (16), we use the term “hold-up effect” somewhat loosely. If $\beta_A \rightarrow 1$ (i.e., if Manager A were residual claimant for his divisional income measure), we would recoup the canonical hold-up model. For general PPS, we need to rescale the marginal cash returns and cost from investing appropriately. This reflects the fact that, unlike most hold-up studies that sidestep compensation issues, our model features both “horizontal” externalities (the γ -sharing of M between the managers) as well as “vertical” externalities (the sharing of π_i among Manager i and the principal as mediated by the PPS).

If $\Delta(I^*(\beta) \mid \beta) > 0$, for a given β , then the hold-up effect dominates and Manager A underinvests, $I^{**}(\beta_A) < I^*(\beta)$. Vice versa, if $\Delta(I^*(\beta) \mid \beta) < 0$, then the risk transfer effect dominates and Manager A overinvests, i.e., $I^{**}(\beta_A) > I^*(\beta)$.

In the following, we present conditions that allow us to evaluate the sign of $\Delta(I^*(\beta) \mid \beta)$, and thereby the nature of the prevailing investment distortion. We first address the standard underinvestment problem:

Condition (UI) $S < S_U$, for some finite and feasible threshold S_U .

The Appendix (proof of Lemma 3) provides a closed-form expression for the threshold S_U invoked in (UI) and shows it is positive and feasible as per (12). Moreover, the S_U -threshold is shown there to be an increasing function of σ_B^2 , so that Condition (UI) restricts the joint project to be only moderately risky relative to the operating uncertainty faced by the non-investing manager, Manager B.

Lemma 3 *If Condition (UI) holds, then Manager A will underinvest, i.e., $I^{**}(\beta_A) < I^*(\beta)$, for any $\beta \in B$.*

Condition (UI) ensures that the risk transfer effect is limited. The severity of the risk transfer effect increases in: (a) the overall incremental project risk resulting from the upfront investment, which is proportional to S , and (b) the extent to which the non-investing party is sensitive to this incremental project risk, as captured by Manager B's PPS. If Manager B faces highly volatile general operations, then he will be relatively insensitive to the additional project risk because the relevant range for his benchmark PPS will entail low-powered incentives (small β_B^{MH}). The hold-up effect then remains the dominant force even if the returns from the joint project are somewhat volatile.²⁶

²⁶The earlier literature that has studied hold-up problems while abstracting from divisional moral hazard problems (e.g., Baldenius, Reichelstein and Sahay, 1999) can be viewed as a limit case of our analysis in which σ_i^2 becomes large. In a highly volatile environment, the PPS remains positive but becomes arbitrarily small. Lemma 3 then applies.

More surprisingly, incomplete contracting may result in overinvestment:

Condition (OI) $S > S_O$, $\sigma_B \in (\sigma_{B,O}^o, \sigma_{B,O}^{oo})$, $\sigma_A > \sigma_{A,O} = \sigma_{B,O}^{oo} + \delta$, $\delta > 0$, and $v < v_O$, for some finite and feasible thresholds $(S_O, \sigma_{B,O}^o, \sigma_{B,O}^{oo}, \sigma_{A,O}, v_O)$.

The Appendix (proof of Lemma 4) provides closed-form expressions for the thresholds invoked in (OI) and shows they are positive and feasible as per (12).

Lemma 4 *If Condition (OI) holds, then Manager A will overinvest, i.e., $I^{**}(\beta_A) > I^*(\beta)$, for any $\beta \in B$.*

Lemma 4 constitutes a departure from the earlier incomplete contracting literature. If the investing manager's environment is more volatile, then the non-investing manager's PPS will be higher-powered, in comparison. This in turn makes the non-investing manager more sensitive to the incremental investment-induced risk. Condition (OI) implies that the incremental project risk is high, and the investing party internalizes only a small portion of the associated risk premium. The additional requirements in Condition (OI) again ensure feasibility as per (12).

Lemmas 3 and 4 take the compensation contracts as given and hence are preliminary in nature. We now show that these results carry over qualitatively to the full-fledged contracting problem. Additional departures from earlier incomplete contracting results obtain, especially regarding the optimal PPS.

5.2 Optimal (Linear) Contracts

Because the principal in our setting can perfectly predict the managers' actions for any given contract, it raises the question, how the principal will adjust the managers' compensation contracts in anticipation of the induced effort and investment choices. We begin by asking how Manager A's investment incentives are affected by the incentive contracts. By (14), Manager B's PPS in our setting does not affect I^{**} , but Manager A's own PPS does:

Proposition 1 *If the investment is non-contractible, $I^{**}(\beta_A)$ is strictly decreasing and convex in the investing manager's PPS, β_A .*

Higher-powered incentives expose Manager A to greater project risk at the margin. As a result, he invests less as his PPS increases. Aside from the usual risk sharing considerations, the investing manager's contract hence trades off his incentives to engage in personally costly effort and investments paid with divisional funds. The convexity part of the result can best be understood by inspecting the investment incentive constraint (14): Manager A's PPS, β_A , scales the project-related incremental risk premium. His investment choice therefore reacts more strongly to a change in β_A for higher levels of I , by Lemma 1. By the “strictly decreasing” part of Proposition 1, higher investment levels go hand in hand with muted incentives, i.e., a low level of β_A .

While Proposition 1 hints at a complex interplay between Manager A's PPS and equilibrium investments, incomplete contracting does not conceptually alter the way Manager B's PPS is determined. Lacking any investment opportunity, Manager B's PPS will always be set so that it trades off effort incentives and risk costs anticipating the respective equilibrium investment levels, i.e., $\beta_B^* = \beta_B^o(I^*)$ in the benchmark setting, and $\beta_B^{**} = \beta_B^o(I^{**})$ under delegation. Using the fact that $\beta_i^o(I)$ is monotonically decreasing in I , the ranking of Manager B's PPS across the two settings is an immediate corollary of the ranking of the equilibrium investments. In fact, Conditions (UI) and (OI) alone predict the investment distortions without the need to fully characterize Manager A's incentive contract:

Proposition 2

- (i) *If Condition (UI) holds, then $I^{**} < I^*$ and, thus, $\beta_B^{**} > \beta_B^*$.*
- (ii) *If Condition (OI) holds, then $I^{**} > I^*$ and, thus, $\beta_B^{**} < \beta_B^*$.*

The predicted investment distortions from Lemmas 3 and 4 carry over to the case of endogenous compensation contracts. The reason is that it is never optimal

for the principal to adjust β_A to the level necessary to induce the benchmark investment I^* . For instance, doing so for the case of (UI) would require muting β_A to the point where the opportunity cost in terms of reduced effort outweighs the investment benefits. Hence, some degree of underinvestment remains in equilibrium. The associated drop in the project risk premium, at the margin, allows the principal to elicit greater effort from Manager B. The arguments for Condition (OI) are analogous.

We now turn to the optimal contract for Manager A, which has to take into account the feedback effect β_A has on equilibrium investments, by Proposition 1. Take again the case of Condition (UI): by Proposition 2, anticipated equilibrium underinvestment would call for raising β_A (we label this the *risk premium effect*). At the same time, Proposition 1 calls for lowering β_A to stimulate investment (the *investment inducement effect*).

To illustrate this tradeoff it is useful to write the principal's expected payoff in reduced form as a function solely of the investing manager's PPS, β_A :

$$\Pi^{**}(\beta_A) \equiv \Pi(\beta_A, \beta_B^o(I^{**}(\beta_A)), I^{**}(\beta_A)).$$

Manager B's PPS will be optimally adjusted to the anticipated equilibrium investment level $I^{**}(\beta_A)$. The distortion in Manager A's PPS is determined by the slope of $\Pi^{**}(\cdot)$ at the benchmark PPS of β_A^* . If $\Pi^{**'}(\beta_A^*) > 0$, then starting from the benchmark level, the principal benefits from raising β_A . Incomplete contracting would then result in higher-powered incentives for Manager A, i.e., $\beta_A^{**} > \beta_A^*$. Vice versa, $\Pi^{**'}(\beta_A^*) < 0$ would imply $\beta_A^{**} < \beta_A^*$. As we show in the Appendix, $\Pi^{**'}(\beta_A^*)$ can be decomposed as follows:

$$\Pi^{**'}(\beta_A^*) = \underbrace{\left(\underbrace{\frac{\partial \Pi(\beta_A, \beta_B^o(I), I)}{\partial I}}_{(X)} \bigg|_{I=I^{**}(\beta_A)} \cdot \underbrace{I^{**'}(\beta_A)}_{(Y)} \right)}_{\text{investment inducement effect}} \bigg|_{\beta_A^*} + \underbrace{\frac{\rho}{4} \beta_A^* \cdot \Delta V(\beta_A^*)}_{\text{risk premium effect, (Z)}}, \quad (17)$$

where, for any β_A ,

$$\Delta V(\beta_A) \equiv \text{Var}(M(\theta, I^*)) - \text{Var}(M(\theta, I^{**}(\beta_A))) \quad (18)$$

denotes the (trade-related) variance differential between the benchmark and the delegation settings.²⁷

The countervailing forces affecting the investing manager's PPS are apparent now: among the three determinants of (17), the investment sensitivity term Y is negative, by Proposition 1. The remaining terms, X (the marginal investment return to the principal) and Z (the marginal risk premium), may take either sign. However, X and Z always have the same sign, which is uniquely determined by the equilibrium investment distortion. Consider the case of (UI), so that $I^{**}(\beta_A^*) \leq I^*$, by Proposition 2. This implies: (a) savings in the differential risk premium at the margin (i.e., $Z \geq 0$) because $\Delta V(\cdot) > 0$; and (b) the principal would benefit from an incremental investment holding fixed the PPS (i.e., $X \geq 0$).²⁸ Therefore, given Condition (UI), an increase in β_A has a negative investment inducement effect (the product of X and Y), but a positive risk premium effect. Conversely, under (OI), both X and Z are negative.

— INSERT TABLE 1 HERE —

²⁷Note that, for the non-investing manager the risk premium effect was the sole determinant of $|\beta_B^{**} - \beta_B^*|$ in Proposition 2 because β_B does not affect investments.

²⁸To illustrate the principal's positive net marginal benefit from greater investment, all else equal, recall that for any I , $\beta_i^o(I) \in \arg \max_{\beta_i} \Pi(I, \beta)$, and $\Pi(I, \beta)$ has decreasing differences, i.e., $\partial \Pi^2 / (\partial I \partial \beta_i) \leq 0$, for all (I, β_i) , $i = A, B$. Therefore, to sign term X in equation (17) for any $I < I^*$: $0 < d\Pi(\beta^o(I), I)/dI = \partial \Pi(\beta^o(I), I)/\partial I < \partial \Pi(\beta_A^*, \beta_B^o(I), I)/\partial I$, where the first inequality uses $I < I^*$, the equality uses the Envelope Theorem, and the second inequality holds because $\beta_A^* < \beta_A^o(I)$ for any $I < I^*$.

We now present sufficient conditions that allow us to evaluate this tradeoff.

Proposition 3 *Given Condition (UI), there exist finite and feasible thresholds $\hat{\sigma}^{oo} \geq \hat{\sigma}^o$ and \hat{f}_U such that:*

- (i) $\beta_A^{**} < \beta_A^*$, if $\sigma_A^2 > \hat{\sigma}^{oo}$;
- (ii) $\beta_A^{**} > \beta_A^*$, if $\sigma_A^2 < \hat{\sigma}^o$ and $f > \hat{f}_U$.

As a result of the countervailing forces on the investing manager's PPS, the directional predictions on β_A^{**} are parameter-specific. Volatile operations at the investing division (Proposition 3(i)) work in favor of the investment inducement effect. For high σ_A^2 , Manager A's PPS will be low, all else equal. This implies: (a) Manager A's investment choice will respond strongly to changes in his PPS (by the convexity result in Proposition 1), i.e., Y is large in absolute terms; (b) the risk premium effect Z is small because it is scaled by β_A^* . The investment inducement effect then is the dominant force, calling for muted incentives for Manager A to alleviate the underinvestment problem.

A relatively stable environment at Division A together with sufficiently convex investment costs (Proposition 3(ii)) work in the opposite direction. The investment inducement effect is dampened, reducing Y in absolute terms, while the risk premium effect Z is scaled up. As a result, β_A is pushed above the benchmark level given (UI).

Turning to the case of (OI), the arguments in the preceding discussion are reversed. Our last result hence is dual in nature to Proposition 3:²⁹

Proposition 3' *Given Condition (OI), there exists a feasible threshold \hat{f}_O such that:*

²⁹The sufficient conditions on σ_A^2 for predicting the distortions in Manager A's PPS are qualitatively similar across the (UI) and (OI) scenarios. The threshold for σ_A^2 is identical across Propositions 3 and 3'. On the other hand, the sufficient condition on the fixed cost function in Proposition 3', as derived in the Appendix, is tighter than that for (UI), i.e., $\hat{f}_O > \hat{f}_U$. This tighter condition ensures existence of $\hat{\sigma}^o$ that satisfies both (12) and (OI).

- (i) $\beta_A^{**} > \beta_A^*$, if $\sigma_A^2 > \hat{\sigma}^{oo}$;
- (ii) $\beta_A^{**} < \beta_A^*$, if $\sigma_A^2 < \hat{\sigma}^o$ and $f > \hat{f}_O$.

With equilibrium *overinvestment*, increasing investment even further would (a) reduce the principal's payoff for given PPS ($X < 0$) and (b) scale up the differential risk premium ($Z < 0$). Volatile operations at Division A again make the investment inducement effect the dominant one (now implying high-powered incentives for Manager A to alleviate the overinvestment tendency). In contrast, moderate σ_A^2 together with high marginal investment costs favor the risk premium effect (now implying muted incentives to adjust for the anticipated expansion in trade). Tables 2 and 3 present a numerical example illustrating our main results. Table 2 shows overinvestment for sufficiently high (yet feasible, by (12)) values of the project uncertainty S . Table 3 varies the operating uncertainty of the investing division to illustrate the equilibrium PPS: the distortions in β_B^{**} are inversely related to the investment distortions, whereas for Manager A this relation is parameter-specific.

— INSERT TABLE 2 AND TABLE 3 HERE —

Our setting with one-sided investment allows us to derive qualitatively different predictions for the incentive contracts offered to managers as a function of their respective scope to invest in joint projects. We find that the PPS for managers without investment opportunities is negatively associated with equilibrium investments undertaken by other managers. As a consequence, if the economic environment is such that underinvestment (overinvestment) is anticipated in equilibrium, incentives for non-investing managers will be higher-powered (lower-powered, respectively) than in the benchmark setting. For managers with investment opportunities, the relation between equilibrium investments and PPS

is ambiguous because of the endogenous nature of investments. Especially in settings where managers' investment choices react sensitively to changes in their PPS (because high general operating risk mutes the PPS to begin with), the equilibrium association between distortions in the PPS and investment levels can be positive.

There is a sizable empirical literature that has focused on regressing the PPS on measures of divisional risk, often however with little regard for synergies across divisions. A separate strand of papers has linked performance measures empirically to the extent of interdivisional interdependencies, e.g., Bushman et al. (1995), Keating (1997), Bouwens and van Lent (2007), Bouwens et al. (2013). These studies focused on “broadening” managers' performance measures by including non-financial metrics or profit sharing. While balanced scorecards and profit sharing are used by many firms, the bulk of division managers' variable compensation appears still to be driven by own-division profit (e.g., Abernethy et al., 2004). Absent such broadened performance measures, the earlier literature had little to say whether or how intrafirm interdependencies should affect a managers' divisional PPS. Our analytical results suggest that such synergies, and especially the relationship-specific investments that support them, can be important omitted variables. More specifically, our results predict systematic differences in managers' PPS as a function of their investment opportunities, even after controlling for their divisional agency problems.

The main mechanism through which investment opportunities in “joint projects” affect the PPS is the equilibrium distortion in specific investments relative to a counterfactual setting in which investments can be directly implemented by the principal. While such investment distortions are generally not observable to the researcher, their underlying conditions, as described by Conditions (UI) and (OI) in our model, are. Take the case of volatile general operations (UI): all else equal, a manager's PPS should be lower if he is given authority to invest than it would

be absent such authority (Proposition 3(i)). To test such predictions, it would be useful to enrich empirical studies on incentives and organizational processes such as Nagar (2002) by distinguishing between the delegation of tasks that are *personally costly* to managers and those which call for managers to invest the *firm's funds* in efficiency-enhancing assets.

6 Concluding Remarks

This article establishes a link between pay-performance sensitivity (PPS) and managers' incentives to invest in joint projects. Greater relationship-specific investments increase the scale of such projects and thus add to the overall compensation risk borne by the managers. Formally accounting for the incremental risk associated with specific investments overturns a number of key results from prior studies. First, compared with the benchmark of contractible investments, a manager always underinvests if, ex post, he holds all bargaining power vis-a-vis his counterpart, but may overinvest for more symmetric bargaining protocols. The reason is that he transfers to the other manager a portion of the investment-induced compensation risk. Second, managers operating under high-powered incentives are reluctant to invest, all else equal. Third, managers facing similar divisional agency problems should receive incentive contracts that vary in a predictable manner in their respective investment opportunities.

The approach taken in most of the prior literature was to look separately at the agency and investment/intrafirm trade problems. This article argues that such a separation hides important linkages between these problems. Accounting for these linkages qualitatively changes several standard results in the incomplete contracting literature. These linkages can be traced to compensation risk that is intrinsically driven by the state uncertainty, which lies at the heart of the incomplete contracting paradigm.

At a more fundamental level, our analysis emphasizes the endogenous nature of risk. Earlier analytical studies have allowed for managers to take actions that directly affect the firm’s risk profile, such as CFOs engaging in risk management (Friedman 2013, 2014). Our article demonstrates that even “standard” investments in operating assets such as PP&E typically have second-moment effects in settings with underlying state uncertainty. While our model has focused on specific investments fostering intrafirm collaborations, managers routinely invest also in equipment aimed at improving the efficiency of external transactions. The issue of endogenous risk, determined jointly with compensation contracts, therefore applies much more broadly than earlier studies may have suggested.

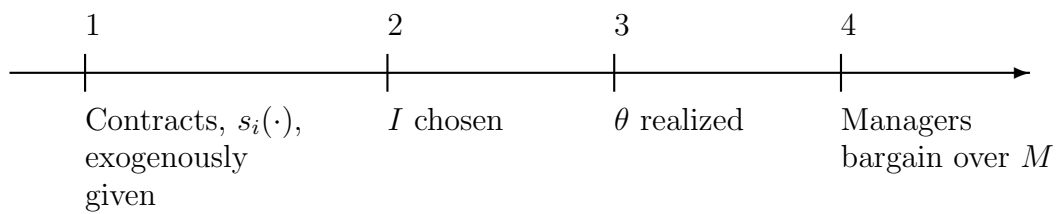


Figure 1: Timeline for Section 2

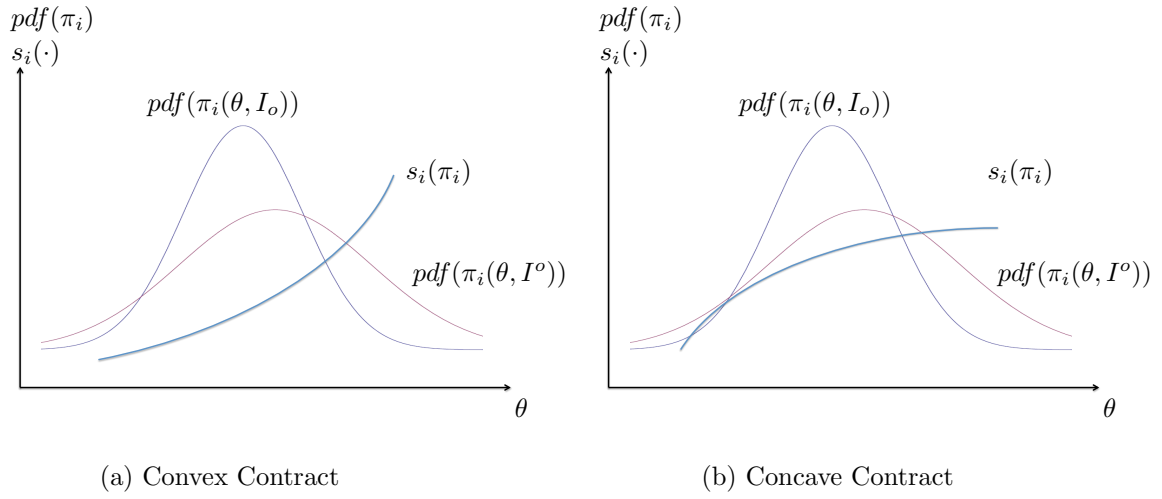


Figure 2: Investment ($I^o > I_o$) increasing the mean and variance of $\pi_i(\theta, I)$

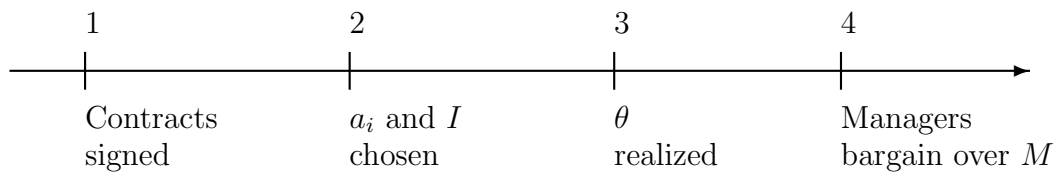


Figure 3: Timeline for Sections 3-5

	$\text{sign}(X)$	$\text{sign}(Y)$	$\text{sign}(Z)$
Condition (UI)	+	−	+
Condition (OI)	−	−	−

Table 1: Summary of signs of X , Y and Z

S	I^*	I^{**}
1.9	10.3047	5.8234
2.0	9.0367	5.2370
2.1	7.7751	4.6518
2.2	6.5200	4.0681
2.3	5.2714	3.4858
2.4	4.0291	2.9048
2.5	2.7931	2.3253
2.6	1.5634	1.7471
2.7	0.3398	1.1702

Table 2: Equilibrium investments

Numerical example: $\mu = 5$, $f = 15$, $\rho = 1$, $v = 0.0001$, $\sigma_B^2 = 24$, $\sigma_A^2 = 4500$.
Here, $S_U = 2$, $S_O = 2.7$ and $\min\{\bar{S}, S_{max}\} = 2.72$. Investments are scaled by 100.

σ_A^2	$\beta_A^* - \beta_A^{**}$	$\beta_B^* - \beta_B^{**}$	$I^* - I^{**}$
5,000	-2.573	-22.170	4.623
7,000	-1.349	-23.937	4.977
9,000	-0.438	-24.650	5.114
11,000	0.197	-24.778	5.133
13,000	0.624	-24.576	5.084
15,000	0.905	-24.189	4.999

(a) Condition (UI) with $S = 1.9$

σ_A^2	$\beta_A^* - \beta_A^{**}$	$\beta_B^* - \beta_B^{**}$	$I^* - I^{**}$
5,000	0.482	4.351	-0.648
7,000	0.070	1.348	-0.200
9,000	0.003	0.269	-0.040
11,000	-0.003	0.256	-0.038
13,000	-0.024	0.838	-0.123
15,000	-0.071	1.748	-0.257

(b) Condition (OI) with $S = 2.7$

Table 3: Equilibrium PPS and investment distortions

Numerical example: $\mu = 5$, $f = 15$, $\rho = 1$, $v = 0.0001$, $\sigma_B^2 = 24$.

PPS are scaled by 10^6 and investments are scaled by 100.

Appendix

Proof of Lemma 1: By definition, $Var(s_i(\pi_i)) = E[s_i(\pi_i)^2] - (E[s_i(\pi_i)])^2$, and hence, using the definition of covariance,

$$\begin{aligned} \frac{\partial Var(s_i(\pi_i(\cdot)))}{\partial I} &= E \left[2s_i(\pi_i(\cdot)) \frac{\partial s_i(\pi_i(\cdot))}{\partial I} \right] - 2E[s_i(\pi_i(\cdot))]E \left[\frac{\partial s_i(\pi_i(\cdot))}{\partial I} \right] \\ &= 2 \cdot Cov \left(s_i(\pi_i(\cdot)), \frac{\partial s_i(\pi_i(\cdot))}{\partial I} \right). \end{aligned}$$

Using results in Schmidt (2003) on the covariance of monotone functions, this term is weakly positive at some arbitrary value $I = x$ if:

$$\begin{aligned} \left. \frac{\partial s_i(\pi_i(\cdot))}{\partial \theta} \right|_{I=x} &= s'_i(\cdot) \gamma_i M_\theta(\cdot) \geq 0, \\ \left. \frac{\partial^2 s_i(\pi_i(\cdot))}{\partial I \partial \theta} \right|_{I=x} &= \frac{\partial}{\partial I} \left(\frac{\partial s_i(\pi_i(\cdot))}{\partial \theta} \right) \\ &= \gamma_i [s''_i(\cdot) M_\theta(\cdot) (\gamma_i M_I(\cdot) - \mathbb{1}_{i=A} F'(x)) + s'_i(\cdot) M_{I\theta}(\cdot)] \\ &\geq 0. \end{aligned} \tag{19}$$

Condition (19) is satisfied under our maintained assumptions. Condition (20) is satisfied if any of the following conditions holds:

- (i) $s''_i(\pi_i) > 0$, for any π_i , and $\left. \frac{\partial \pi_i(\theta, I)}{\partial I} \right|_{I=x} = \gamma_i M_I(\cdot) - \mathbb{1}_{i=A} F'(x) \geq 0$, for any θ ;
- (ii) $s''_i(\pi_i) < 0$, for any π_i , and $\left. \frac{\partial \pi_i(\theta, I)}{\partial I} \right|_{I=x} = \gamma_i M_I(\cdot) - \mathbb{1}_{i=A} F'(x) \leq 0$, for any θ ;
- (iii) $|s''_i(\pi_i)| < \delta$, for any π_i and some δ positive but sufficiently small.

■

Calculation of the variance of the contribution margin: The expectation of the contribution margin, $M(\theta, I) = \frac{(\theta+I)^2}{2}$, is:

$$E[M(\theta, I)] = \frac{1}{2} \left(\frac{(\mu - \sqrt{S} + I)^2}{2} \right) + \frac{1}{2} \left(\frac{(\mu + \sqrt{S} + I)^2}{2} \right) = \frac{(\mu + I)^2 + S}{2}.$$

The variance of the contribution margin, $M(\theta, I) = \frac{(\theta+I)^2}{2}$, follows as:

$$\begin{aligned}
Var(M(\theta, I)) &= \frac{1}{2} \left(\frac{(\mu - \sqrt{S} + I)^2}{2} - \frac{(\mu + I)^2 + S}{2} \right)^2 \\
&\quad + \frac{1}{2} \left(\frac{(\mu + \sqrt{S} + I)^2}{2} - \frac{(\mu + I)^2 + S}{2} \right)^2 \\
&= (\mu + I)^2 S \\
&= [q^*(\mu, I)]^2 S.
\end{aligned} \tag{21}$$

■

Proof of Lemma 3: As defined in the main text, let $I^*(\beta) \in \arg \max_I \Pi(I, \beta)$ for any $\beta \in B \equiv [\beta_A^{min}, \beta_A^{MH}] \times [\beta_B^{min}, \beta_B^{MH}]$. The first-order condition for the optimal contractible benchmark investment level, in (15), is:

$$\left. \frac{\partial}{\partial I} \Pi(I, \beta) \right|_{I^*(\beta)} = q^*(\mu, I^*) - fI^* - \frac{\rho S}{4}(\beta_A^2 + \beta_B^2)q^*(\mu, I^*) = 0. \tag{22}$$

Our maintained assumption that $f > 1$ ensures concavity. For given $\beta_A \in [\beta_A^{min}, \beta_A^{MH}]$, using (21), Manager A sets the non-contractible investment $I^{**}(\beta_A)$ such that

$$\Gamma'(I \mid \beta_A) = \beta_A \left(\frac{q^*(\mu, I)}{2} - fI - \frac{\rho S}{4}\beta_A q^*(\mu, I) \right)$$

equals zero at $I = I^{**}(\beta_A)$. To compare $I^{**}(\beta_A)$ with $I^*(\beta)$, it is sufficient to sign the derivative $\Gamma'(I \mid \beta_A)$ at $I = I^*(\beta)$. Using (22),

$$\begin{aligned}
\Gamma'(I \mid \beta_A)|_{I^*(\beta)} &= \beta_A \left(\frac{\rho S}{4}(\beta_B^2 + \beta_A^2 - \beta_A)q^*(\mu, I^*) - \frac{q^*(\mu, I^*)}{2} \right) \\
&\propto \frac{\rho S}{2}[\beta_B^2 - \beta_A(1 - \beta_A)] - 1 \\
&\equiv h.
\end{aligned}$$

A necessary and sufficient condition for $I^{**}(\beta_A) < I^*(\beta)$ therefore is that $h < 0$. First, note that $\beta_A(1 - \beta_A) \in (0, \frac{1}{4})$ for any $\beta_A \in (\beta_A^{min}, \beta_A^{MH})$. Moreover,

because $\beta_B < \beta_B^{MH} = (1 + \rho v \sigma_B^2)^{-1}$:

$$h \equiv \frac{\rho S}{2} [\beta_B^2 - \beta_A(1 - \beta_A)] - 1 < \frac{\rho S}{2} \beta_B^2 - 1 < \frac{\rho S}{2} (\beta_B^{MH})^2 - 1 < 0, \quad (23)$$

if $S < S_U \equiv \frac{2}{\rho} (1 + \rho v \sigma_B^2)^2$. In summary, $I^*(\beta) > I^{**}(\beta_A)$ for any $\beta \in B$ if $S < S_U$, i.e., if Condition (UI) holds. \blacksquare

Proof of Lemma 4: Building on the proof of Lemma 3, $I^{**}(\beta_A) > I^*(\beta)$ if and only if $h \equiv \frac{\rho S}{2} [\beta_B^2 - \beta_A(1 - \beta_A)] - 1 > 0$. We derive sufficient conditions for this to be the case and then verify that these conditions satisfy the feasibility requirement in (12). A necessary condition for $h > 0$ is that $(\beta_B)^2 > \beta_A(1 - \beta_A)$. Using $\beta_A(1 - \beta_A) < \frac{1}{4}$, and $\beta_B \geq \beta_B^{min}$, we have that $(\beta_B)^2 > \beta_A(1 - \beta_A)$ if

$$\sigma_B^2 < \frac{1}{2\rho v}. \quad (24)$$

Lastly, given that $(\beta_B)^2 > \beta_A(1 - \beta_A)$, $h > 0$ if

$$S > S_O \equiv \frac{2}{\rho ((\beta_B^{min})^2 - \frac{1}{4})} = \frac{2}{\rho \left(\frac{1}{(1+2\rho v \sigma_B^2)^2} - \frac{1}{4} \right)}. \quad (25)$$

It remains to verify feasibility of (25) as per (12) under the stated conditions.

We first confirm that $S_O < \bar{S}$:

$$\begin{aligned} S_O - \bar{S} &= \frac{2}{\rho ((\beta_B^{min})^2 - \frac{1}{4})} - \frac{4}{\rho \sum_i (\beta_i^{MH})^2} \\ &\propto \sum_i (\beta_i^{MH})^2 - 2(\beta_B^{min})^2 + \frac{1}{2}. \end{aligned}$$

A sufficient condition for this term to be negative is that $\sigma_B < \sigma_{B,O}^{oo} \equiv \frac{0.05}{\sqrt{\rho v}}$ and $\sigma_A > \sigma_{B,O}^{oo} + \delta$, where $\delta \equiv \frac{0.61}{\sqrt{\rho v}}$. It remains to confirm that $S_O < S_{max}$:

$$\begin{aligned} S_O - S_{max} &= \frac{2}{\rho ((\beta_B^{min})^2 - \frac{1}{4})} - \frac{4}{q^*(\mu, \hat{I})^2} \min\{\sigma_A^2, \sigma_B^2\} \\ &\propto q^*(\mu, \hat{I})^2 - 2\rho \sigma_B^2 \left((\beta_B^{min})^2 - \frac{1}{4} \right) \\ &\leq \left(\frac{\mu f}{f-1} \right)^2 - \frac{15}{10} \rho \sigma_B^2, \end{aligned}$$

because $\sigma_B < \sigma_{B,O}^{oo}$. The last term is negative if $\sigma_B^2 > (\sigma_{B,O}^o)^2 \equiv \frac{10}{15\rho} \left(\frac{\mu f}{f-1} \right)^2$. This is feasible if $(\sigma_{B,O}^o)^2 < (\sigma_{B,O}^{oo})^2$ which holds if $v < v_O \equiv \frac{3}{800} \left(\frac{f-1}{\mu f} \right)^2$.

We have shown that (25) is feasible as per (12) if $v < v_O$, $\sigma_B \in (\sigma_{B,O}^o, \sigma_{B,O}^{oo})$, and $\sigma_A > \sigma_{A,O} = \sigma_{B,O}^{oo} + \delta$, $\delta > 0$. We note that (24) holds provided $\sigma_B < \sigma_{B,O}^{oo}$. Combining (24), (25), and the feasibility conditions above, we conclude that

$$h \equiv \frac{\rho S}{2} [\beta_B^2 - \beta_A(1 - \beta_A)] - 1 > 0, \quad \forall \beta \in B, \quad (26)$$

if $S > S_O$, $v < v_O$, $\sigma_B \in (\sigma_{B,O}^o, \sigma_{B,O}^{oo})$, and $\sigma_A > \sigma_{A,O} = \sigma_{B,O}^{oo} + \delta$, $\delta > 0$, i.e., if Condition (OI) holds. ■

Proof of Proposition 1: Applying the Implicit Function Theorem to (14),

$$\begin{aligned} I^{**'}(\beta_A) &= - \frac{\frac{q^*(\mu, I^{**}(\beta_A))}{2} - f I^{**}(\beta_A) - \frac{\rho S}{2} \beta_A q^*(\mu, I^{**}(\beta_A))}{\frac{\beta_A}{2} - \beta_A f - \frac{\rho S}{4} \beta_A^2} \\ &= \frac{\frac{\rho S}{4} \beta_A q^*(\mu, I^{**}(\beta_A))}{\frac{\beta_A}{2} - \beta_A f - \frac{\rho S}{4} \beta_A^2} \end{aligned} \quad (27)$$

$$= - \frac{\frac{\rho S}{4} q^*(\mu, I^{**}(\beta_A))}{\frac{\rho S}{4} \beta_A + f - \frac{1}{2}} < 0, \quad (28)$$

where (27) uses (14), while the inequality in (28) uses the fact that $f > 1$. To prove convexity, we use $q^*(\mu, I^{**}(\beta_A)) = \mu + I^{**}(\beta_A)$ and (28),

$$\begin{aligned} I^{**''}(\beta_A) &= - \frac{\partial}{\partial \beta_A} \left(\frac{\frac{\rho S}{4} q^*(\mu, I^{**}(\beta_A))}{\frac{\rho S}{4} \beta_A + f - \frac{1}{2}} \right) \\ &= - \left(\frac{\frac{\rho S}{4} I^{**'}(\beta_A)}{\frac{\rho S}{4} \beta_A + f - \frac{1}{2}} - \frac{\left(\frac{\rho S}{4} \right)^2 q^*(\mu, I^{**}(\beta_A))}{\left(\frac{\rho S}{4} \beta_A + f - \frac{1}{2} \right)^2} \right) \\ &= 2 \frac{\left(\frac{\rho S}{4} \right)^2 q^*(\mu, I^{**}(\beta_A))}{\left(\frac{\rho S}{4} \beta_A + f - \frac{1}{2} \right)^2} > 0. \end{aligned}$$

■

Proof of Proposition 2

Part (i): Given Condition (UI), by Lemma 3, $I^{**}(\beta_A^*) < I^*$ because $\beta_A^* \in [\beta_A^{min}, \beta_A^{MH}]$. By Proposition 1, to induce greater investment, the principal has to

set $\beta_A < \beta_A^*$. If $\lim_{\beta_A \rightarrow 0} I^{**}(\beta_A) < I^*$, then $I^{**} < I^*$ holds. If $\lim_{\beta_A \rightarrow 0} I^{**}(\beta_A) > I^*$, then there exists a unique threshold PPS, $\underline{\beta} \in (0, \beta_A^*)$, such that $I^{**}(\underline{\beta}) \equiv I^*$. Define the value function

$$\Pi^{**}(\beta_A) \equiv \Pi(\beta_A, \beta_B^o(I^{**}(\beta_A)), I^{**}(\beta_A)).$$

The proof strategy entails showing that, if such a positive $\underline{\beta}$ exists, the principal would benefit from setting $\beta_A > \underline{\beta}$, i.e., $\Pi^{**'}(\underline{\beta}) > 0$.

Because the principal chooses β_B optimally conditional on the non-contractible investment $I^{**}(\beta_A)$, by the Envelope Theorem,

$$\Pi^{**'}(\beta_A) = \frac{\partial \Pi(\beta_A, \beta_B^o(I^{**}(\beta_A)), I^{**}(\beta_A))}{\partial \beta_A} + \frac{\partial \Pi(\beta_A, \beta_B^o(I^{**}(\beta_A)), I)}{\partial I} \Big|_{I=I^{**}(\beta_A)} I^{**'}(\beta_A). \quad (29)$$

By definition, $\Pi^{**'}(\beta_A^{**}) = 0$. Applying our maintained assumption that $f > 1$ to (28), yields a lower bound on $I^{**'}(\beta_A)$:

$$I^{**'}(\beta_A) > -\frac{q^*(\mu, I^{**}(\beta_A))}{\beta_A}.$$

We then have:

$$\begin{aligned}\Pi^{**'}(\underline{\beta}) &= \left(q^*(\mu, I^*) - F'(I^*) - \frac{\rho}{8}(\underline{\beta}^2 + (\beta_B^*)^2) \frac{\partial Var(M(\theta, I^*))}{\partial I} \right) I^{**'}(\underline{\beta}) \\ &\quad + \Phi'_A(\underline{\beta}) - \frac{\rho}{4}\underline{\beta} Var(M(\theta, I^*))\end{aligned}\quad (30)$$

$$= -\frac{\rho}{8}(\underline{\beta}^2 - (\beta_A^*)^2) \frac{\partial Var(\cdot)}{\partial I} \cdot I^{**'}(\underline{\beta}) + \Phi'_A(\underline{\beta}) - \frac{\rho}{4}\underline{\beta} Var(\cdot) \quad (31)$$

$$> \frac{\rho}{8}(\underline{\beta}^2 - (\beta_A^*)^2) \frac{\partial Var(\cdot)}{\partial I} \cdot \frac{q^*(\mu, I^*)}{\underline{\beta}} + \Phi'_A(\underline{\beta}) - \frac{\rho}{4}\underline{\beta} Var(\cdot) \quad (32)$$

$$\begin{aligned}&= \frac{\rho}{4} \left(\underline{\beta} - \frac{(\beta_A^*)^2}{\underline{\beta}} \right) (q^*(\mu, I^*))^2 S + \Phi'_A(\underline{\beta}) - \frac{\rho}{4}\underline{\beta} Var(\cdot) \\ &= \frac{\rho}{4} \left(\underline{\beta} - \frac{(\beta_A^*)^2}{\underline{\beta}} \right) Var(M(\theta, I^*)) + \Phi'_A(\underline{\beta}) - \frac{\rho}{4}\underline{\beta} Var(\cdot)\end{aligned}\quad (33)$$

$$\begin{aligned}&= \Phi'_A(\underline{\beta}) - \frac{\rho}{4} \frac{(\beta_A^*)^2}{\underline{\beta}} Var(M(\theta, I^*)) \\ &> \Phi'_A(\underline{\beta}) - \frac{\rho}{4} \beta_A^* Var(M(\theta, I^*))\end{aligned}\quad (34)$$

$$= \Phi'_A(\underline{\beta}) - \Phi'_A(\beta_A^*) \quad (35)$$

$$> 0. \quad (36)$$

Here, (30) uses $I^{**}(\underline{\beta}) = I^*$ and $\beta_B^o(I^{**}(\underline{\beta})) = \beta_B^*$; (31) uses the first-order condition for contractible investment in (22); (32) uses the above lower bound on $I^{**'}(\beta_A)$; (33) uses (21); (34) uses $\beta_A^* > \underline{\beta}$; (35) uses the first-order condition for contractible PPS, $\Phi'_i(\beta_i^*) = \frac{\rho}{4}\beta_i^* Var(M(\theta, I^*))$; and (36) uses $\Phi'_i(0) > \Phi'_i(\beta_i^*) > \Phi'_i(\beta_i^{MH}) = 0 > \Phi'_i(1)$, $\Phi''_i(\cdot) \leq 0$, and $\underline{\beta} < \beta_A^*$. It follows that $\beta_A^{**} > \underline{\beta}$, and, hence, $I^{**} \equiv I^{**}(\beta_A^{**}) < I^*$. Thus, $\beta_B^{**} \equiv \beta_B^o(I^{**}) > \beta_B^* \equiv \beta_B^o(I^*)$, because $\beta_i^o(I)$, $i = A, B$, is a decreasing function.

Part (ii): By Lemma 4, if Condition (OI) holds, $I^{**}(\beta_A^*) > I^*$ because $\beta_A^* \in [\beta_A^{min}, \beta_A^{MH}]$. By Proposition 1, to dampen investment incentives, the principal has to increase β_A above the contractible benchmark. Because $\lim_{\beta_A \rightarrow \infty} I^{**}(\beta_A) = 0$, by Proposition 1, there exists a unique $\bar{\beta} > \beta_A^*$, such that $I^{**}(\bar{\beta}) = I^*$. To compare $\bar{\beta}$ and β_A^{**} , we need to sign $\Pi^{**'}(\bar{\beta})$. The remainder of the proof mirrors

the proof of Proposition 2(i), and hence is omitted. \blacksquare

Proof of Proposition 3: Comparing β_A^{**} and β_A^* requires signing $\Pi^{**'}(\beta_A^*)$. To avoid clutter, let $\tilde{\beta}_B \equiv \beta_B^o(I^{**}(\beta_A^*))$. Using (29),

$$\begin{aligned} \Pi^{**'}(\beta_A^*) &= \left. \frac{\partial \Pi(\beta_A^*, \beta_B^o(I^{**}(\beta_A^*)), I)}{\partial I} \right|_{I=I^{**}(\beta_A^*)} \cdot I^{**'}(\beta_A^*) + \Phi'_A(\beta_A^*) \\ &\quad - \frac{\rho}{4} \beta_A^* \text{Var}(M(\theta, I^{**}(\beta_A^*))). \end{aligned} \quad (37)$$

We start by evaluating the first term in (37):

$$\begin{aligned} \left. \frac{\partial \Pi(\cdot)}{\partial I} \right|_{I=I^{**}(\beta_A^*)} &= q^*(\mu, I^{**}(\beta_A^*)) - f I^{**}(\beta_A^*) - \frac{\rho}{8} ((\beta_A^*)^2 + \tilde{\beta}_B^2) \frac{\partial \text{Var}(M(\theta, I^{**}(\beta_A^*)))}{\partial I} \\ &= \frac{q^*(\mu, I^{**}(\beta_A^*))}{2} - \frac{\rho}{8} ((\beta_A^*)^2 + \tilde{\beta}_B^2 - \beta_A^*) \frac{\partial \text{Var}(M(\theta, I^{**}(\beta_A^*)))}{\partial I} \end{aligned} \quad (38)$$

$$= -\frac{q^*(\mu, I^{**}(\beta_A^*))}{2} \left[\frac{\rho S}{2} ((\beta_A^*)^2 + \tilde{\beta}_B^2 - \beta_A^*) - 1 \right] \quad (39)$$

where (38) uses (13) and (39) uses (21). Next, we evaluate the last two terms in (37):

$$\Phi'_A(\beta_A^*) - \frac{\rho}{4} \beta_A^* \text{Var}(M(\theta, I^{**}(\beta_A^*))) = \frac{\rho}{4} \beta_A^* \cdot (\Delta V(\beta_A^*)) \quad (40)$$

$$= \frac{\rho S}{4} \beta_A^* ([q^*(\mu, I^*)]^2 - [q^*(\mu, I^{**}(\beta_A^*))]^2), \quad (41)$$

where (40) uses (18) and the first-order condition for the contractible PPS,

$\Phi'_A(\beta_A^*) = \frac{\rho}{4} \beta_A^* \text{Var}(M(\theta, I^*))$; and (41) uses (21). Next, using (21) and $q^*(\mu, I) =$

$\mu + I$, I^* and $I^{**}(\beta_A^*)$ satisfy, respectively,

$$q^*(\mu, I^*) - f I^* - \frac{\rho S}{4} \sum_i (\beta_i^*)^2 q^*(\mu, I^*) = 0 \iff I^* = \frac{\mu(1 - \frac{\rho S}{4} \sum_i (\beta_i^*)^2)}{\frac{\rho S}{4} \sum_i (\beta_i^*)^2 + f - 1},$$

$$\frac{q^*(\mu, I^{**}(\beta_A^*))}{2} - f I^{**}(\beta_A^*) - \frac{\rho S}{4} \beta_A^* q^*(\mu, I^{**}(\beta_A^*)) = 0 \iff I^{**}(\beta_A^*) = \frac{\frac{\mu}{2}(1 - \frac{\rho S}{2} \beta_A^*)}{\frac{\rho S}{4} \beta_A^* + f - \frac{1}{2}}.$$

Hence,

$$q^*(\mu, I^*) = \mu + \frac{\mu(1 - \frac{\rho S}{4} \sum_i (\beta_i^*)^2)}{\frac{\rho S}{4} \sum_i (\beta_i^*)^2 + f - 1} \quad (42)$$

$$q^*(\mu, I^{**}(\beta_A^*)) = \mu + \frac{\frac{\mu}{2}(1 - \frac{\rho S}{2} \beta_A^*)}{\frac{\rho S}{4} \beta_A^* + f - \frac{1}{2}}. \quad (43)$$

Then, substituting (28), (39), (41), (42) and (43) into (37) and simplifying:

$$\begin{aligned}
\Pi^{**'}(\beta_A^*) &= \frac{\rho S [q^*(\mu, I^{**}(\beta_A^*))]^2}{8(\frac{\rho S}{4}\beta_A^* + f - \frac{1}{2})} \left(\frac{\rho S}{2} ((\beta_A^*)^2 + \tilde{\beta}_B^2 - \beta_A^*) - 1 \right) \\
&\quad + \frac{\rho S}{4} \beta_A^* ([q^*(\mu, I^*)]^2 - [q^*(\mu, I^{**}(\beta_A^*))]^2) \\
&\propto \frac{\rho S}{2} [(\beta_A^*)^2 + \tilde{\beta}_B^2 - \beta_A^*] - 1 + 2\beta_A^* \left(\left(\frac{q^*(\mu, I^*)}{q^*(\mu, I^{**}(\beta_A^*))} \right)^2 - 1 \right) \left(\frac{\rho S}{4}\beta_A^* + f - \frac{1}{2} \right) \\
&= \frac{\rho S}{2} [(\beta_A^*)^2 + \tilde{\beta}_B^2 - \beta_A^*] - 1 \\
&\quad - \frac{\beta_A^* \left(\frac{\rho S}{2} (\sum_i (\beta_i^*)^2 - \beta_A^*) - 1 \right) \left(\frac{\rho S}{4} (\sum_i (\beta_i^*)^2 + \beta_A^*) + 2f - \frac{3}{2} \right) \left(\frac{\rho S}{4}\beta_A^* + f - \frac{1}{2} \right)}{\left(\frac{\rho S}{4} \sum_i (\beta_i^*)^2 + f - 1 \right)^2}.
\end{aligned}$$

Using (23) and $\beta_i^* \in [\beta_i^{min}, \beta_i^{MH}]$, this term is proportional to:

$$\begin{aligned}
K &\equiv \beta_A^* \underbrace{\left(\frac{\rho S}{4} (\sum_i (\beta_i^*)^2 + \beta_A^*) + 2f - \frac{3}{2} \right) \left(\frac{\rho S}{4}\beta_A^* + f - \frac{1}{2} \right)}_{\equiv H_1} \\
&\quad - \left(\frac{\rho S}{4} \sum_i (\beta_i^*)^2 + f - 1 \right)^2 \underbrace{\left(\frac{\frac{\rho S}{2} [(\beta_A^*)^2 + \tilde{\beta}_B^2 - \beta_A^*] - 1}{\frac{\rho S}{2} (\sum_i (\beta_i^*)^2 - \beta_A^*) - 1} \right)}_{\equiv H_2}. \quad (44)
\end{aligned}$$

A necessary and sufficient condition for $\beta_A^{**} < \beta_A^*$ (respectively, $\beta_A^{**} > \beta_A^*$) is that $K < 0$ (respectively, $K > 0$).

Part (i): We derive sufficient conditions for $K < 0$ (so that $\beta_A^{**} < \beta_A^*$). First, $\left(\frac{3\rho S}{4} + 2f - \frac{3}{2} \right) \left(\frac{\rho S}{4} + f - \frac{1}{2} \right) > H_1 > 0$ because $\beta_i^* \in (0, 1)$. Second,

$$H_2 > \frac{\frac{\rho S}{2} [(\beta_B^{min})^2 - \frac{1}{4}] - 1}{\frac{\rho S}{2} (\beta_B^{MH})^2 - 1} = \frac{\frac{\rho S}{2} \left[\frac{1}{(1+2\rho v \sigma_B^2)^2} - \frac{1}{4} \right] - 1}{\frac{\rho S}{2(1+\rho v \sigma_B^2)^2} - 1} \equiv \gamma, \quad (45)$$

because $\beta_B^* < \beta_B^{MH}$; $\tilde{\beta}_B > \beta_B^{min}$ and $\beta_A^* (\beta_A^* - 1) \in (-\frac{1}{4}, 0)$. Third, $\gamma > 0$ because $\frac{\rho S}{2} [(\beta_B^{min})^2 - \frac{1}{4}] - 1 < \frac{\rho S}{2} (\beta_B^{MH})^2 - 1 < 0$ by (23). Fourth, $\beta_A^* < \beta_A^{MH}$. Hence,

$$\begin{aligned}
K &< \beta_A^{MH} \left(\frac{3\rho S}{4} + 2f - \frac{3}{2} \right) \left(\frac{\rho S}{4} + f - \frac{1}{2} \right) - (f-1)^2 \gamma \\
&\equiv k_1.
\end{aligned}$$

Using the definition of β_i^{MH} , $k_1 < 0$ (and hence $\Pi^{**'}(\beta_A^*) < 0$ and $\beta_A^{**} < \beta_A^*$) if

$$\sigma_A^2 > \hat{\sigma}^{oo} \equiv \frac{\left(\frac{3\rho S}{4} + 2f - \frac{3}{2}\right) \left(\frac{\rho S}{4} + f - \frac{1}{2}\right) - (f-1)^2\gamma}{\rho v(f-1)^2\gamma}.$$

Note that $\hat{\sigma}^{oo} > 0$, because $f > 1$ and $\gamma \in (0, 1)$.

Part (ii): We now derive sufficient conditions for $K > 0$ (so that $\beta_A^{**} > \beta_A^*$):

$$K > \beta_A^{min} \left(2f - \frac{3}{2}\right) \left(f - \frac{1}{2}\right) - \left(\frac{\rho \bar{S}}{4} \sum_i (\beta_i^*)^2 + f - 1\right)^2 \quad (46)$$

$$\begin{aligned} &> \beta_A^{min} \left(2f - \frac{3}{2}\right) \left(f - \frac{1}{2}\right) - f^2 \\ &\equiv k_2 \end{aligned} \quad (47)$$

where inequality (46) uses (a) $H_1 > \left(2f - \frac{3}{2}\right) \left(f - \frac{1}{2}\right) - \left(\frac{\rho S}{4} \sum_i (\beta_i^*)^2 + f - 1\right)^2 > 0$; (b) $\beta_A^{min} < \beta_A^*$; (c) $H_2 < \frac{\frac{\rho S}{2}(\sum_i (\beta_i^*)^2 - \beta_A^*) - 1}{\frac{\rho S}{2}(\sum_i (\beta_i^*)^2 - \beta_A^*) - 1} = 1$ because $\beta_B^* < \tilde{\beta}_B$ by $\tilde{\beta}_B \equiv \beta_B^o(I^{**}(\beta_A^*))$, Lemma 3 and (23); and (d) $S < \bar{S}$. Inequality (47) uses the definition of \bar{S} . Using the definition of β_i^{min} , $k_2 > 0$ (and hence $\Pi^{**'}(\beta_A^*) > 0$ and $\beta_A^{**} > \beta_A^*$) if

$$\sigma_A^2 < \hat{\sigma}^o \equiv \frac{\left(2f - \frac{3}{2}\right) \left(f - \frac{1}{2}\right) - f^2}{2\rho v f^2}.$$

This is feasible, i.e. $\hat{\sigma}^o > 0$, if $f > \hat{f}_U = \frac{1}{4}(5 + \sqrt{13}) \approx 2.15$. Lastly, we note that $\hat{\sigma}^o < \hat{\sigma}^{oo}$ because $k_1 < K < k_2$ for any σ_A^2 . ■

Proof of Proposition 3': The comparison of β_A^{**} and β_A^* follows similar steps as the proof of Proposition 3. The only difference here is that, under (OI), $\frac{\rho S}{2} [\beta_A^2 + \beta_B^2 - \beta_A] - 1 > 0$ for $\beta \in [\beta_A^{min}, \beta_A^{MH}] \times [\beta_B^{min}, \beta_B^{MH}]$ by (26) and therefore we find that $\Pi^{**'}(\beta_A^*) \propto -K$, where K is as defined in (44).

Part (i): The proof of this part follows analogous steps as the ones in the proof of Proposition 3(i). It is straightforward to see that $-K > -k_1 > 0$ (and hence $\Pi^{**'}(\beta_A^*) > 0$ and $\beta_A^{**} > \beta_A^*$) if

$$\sigma_A^2 > \hat{\sigma}^{oo} \equiv \frac{\left(\frac{3\rho S}{4} + 2f - \frac{3}{2}\right) \left(\frac{\rho S}{4} + f - \frac{1}{2}\right) - (f-1)^2\gamma}{\rho v(f-1)^2\gamma},$$

where γ as defined in (45) is positive, because $0 < \frac{\rho^S}{2} [(\beta_B^{min})^2 - \frac{1}{4}] - 1 < \frac{\rho^S}{2} (\beta_B^{MH})^2 - 1$ by (26).

Part (ii): The proof of this part follows analogous steps as the ones in the proof of Proposition 3(ii). It is straightforward to see that $-K < -k_2 < 0$ (and hence $\Pi^{**'}(\beta_A^*) > 0$ and $\beta_A^{**} > \beta_A^*$) if

$$\sigma_A^2 < \hat{\sigma}^o \equiv \frac{(2f - \frac{3}{2})(f - \frac{1}{2}) - f^2}{2\rho v f^2}.$$

This is feasible by (12) and Condition (OI), i.e., $\sigma_{A,O}^2 < \hat{\sigma}^o$, if $f > \hat{f}_O = \frac{25}{322} (125 + \sqrt{14,659}) \approx 19.11$. (Note that the lower bound on f , $f > \hat{f}_O$, is a sufficient condition for $\beta_A^{**} > \beta_A^*$. It is not necessary, as illustrated by the numerical example in Table 2.) Lastly, we note that $\hat{\sigma}^o < \hat{\sigma}^{oo}$ because $-k_1 < -K < -k_2$ for any σ_A^2 . ■

References

- [1] Abernethy, M., J. Bouwens, and L. van Lent. 2004. Determinants of Control System Design in Divisionalized Firms. *The Accounting Review* 79(3): 545-570.
- [2] Anctil, R., and S. Dutta. 1999. Negotiated Transfer Pricing and Divisional vs. Firm-Wide Performance Evaluation. *The Accounting Review* 74(1): 87-104.
- [3] Armstrong, C., D. Larcker, G. Ormazabal, and D. Taylor. 2013. The Relation Between Equity Incentives and Misreporting: The Role of Risk-Taking Incentives. *Journal of Financial Economics* 109: 327-350.
- [4] Arya, A., and B. Mittendorf. 2010. Input Markets and the Strategic Organization of the Firm. *Foundations and Trends in Accounting* 5(1): 1-97.
- [5] Baiman, S., and T. Baldenius, 2009. Non-Financial Performance Measures as Coordination Devices. *The Accounting Review* 84(2): 299-330.
- [6] Baldenius, T., and S. Reichelstein, 2006. External and Internal Pricing in Multidivisional Firms. *Journal of Accounting Research* 44(1): 1-28.
- [7] Baldenius, T., S. Reichelstein, and S. Sahay. 1999. Negotiated versus Cost-Based Transfer Pricing. *Review of Accounting Studies* 4(2): 67-91.
- [8] Baker, G., and B. Jorgensen. 2003. *Volatility, Noise, and Incentives*. Working paper.
- [9] Bertomeu, J. 2008. *Risk Management, Executive Compensation and the Cross-Section of Corporate Earnings*. Working paper.
- [10] Bouwens, J., and L. van Lent. 2007. Assessing the Performance of Business Unit Managers. *Journal of Accounting Research* 45(4): 667-697.

- [11] Bouwens, J., C. Hofmann, and L. van Lent. 2013. *Performance Measures and Intra-Firm Spillovers: Theory and Evidence*. Working paper.
- [12] Bushman, R., R. Indjejikian, and A. Smith. 1995. Aggregate Performance Measures in Business Unit Manager Compensation: The Role of Intrafirm Interdependencies. *Journal of Accounting Research* 33: 101-128.
- [13] Carpenter, J. 2000. Does Option Compensation Increase Managerial Risk Appetite? *Journal of Finance* 55: 2311-2331.
- [14] Chung, T.-Y. 1991. Incomplete Contracts, Specific Investments, and Risk Sharing. *Review of Economic Studies* 58(5): 1031-1042.
- [15] Christensen, P., F. Sabac, and J. Tian. 2010. Ranking Performance Measures in Multitask Agencies. *The Accounting Review* 85(5): 1545-1575.
- [16] Edlin, A., and S. Reichelstein. 1995. Specific Investment under Negotiated Transfer Pricing: An Efficiency Result. *The Accounting Review* 70: 275-291.
- [17] Feltham, G., and D. Xie. 1994. Performance Measure Congruity and Diversity in Multitask Principal-Agent Relations. *The Accounting Review* 69: 429-454.
- [18] Friedman, H. 2013. *The Role and Responsibilities of the CFO: Implications for Reporting Quality, Firm Value, and Incentive Compensation*. Working paper.
- [19] Friedman, H. 2014. Implications of Power: When the CEO Can Pressure the CFO to Bias Reports. *Journal of Accounting and Economics* 58: 117-141.
- [20] Göx, R., and U. Schiller. 2007. An Economic Perspective on Transfer Pricing. In *Handbook of Management Accounting Research*, edited by C. S. Chapman, A. G. Hopwood, and M. D. Shields.

- [21] Heinle, M., C. Hofmann and A. Kunz. 2012. Identity, Incentives, and the Value of Information. *The Accounting Review* 87(4): 1309-1334.
- [22] Holmstrom, B., and P. Milgrom. 1991. Multitask Principal-Agent Analysis: Incentive Contracts, Asset Ownership, and Job Design. *Journal of Law, Economics, & Organization* 7: 24-52.
- [23] Holmstrom, B., and J. Tirole. 1991. Transfer Pricing and Organizational Form. *Journal of Law, Economics and Organization* 7 (2): 201-228.
- [24] Indjejikian, R., and M. Matejka. 2009. CFO Fiduciary Responsibilities and Annual Bonus Incentives. *Journal of Accounting Research* 47(4): 1061-1093.
- [25] Liang, P., and L. Nan. 2014. Endogenous Precision of Performance Measures and Limited Managerial Attention. *European Accounting Review* 23(4): 693-727
- [26] Liang, P., M. Rajan, and K. Ray. 2008. Optimal Team Size and Monitoring in Organizations. *The Accounting Review* 83(3): 789-822.
- [27] Melumad, N., D. Mookherjee, and S Reichelstein. 1992. A Theory of Responsibility Centers. *Journal of Accounting and Economics* 15 (4): 445-84.
- [28] Nagar, V. 2002. Delegation and Incentive Compensation. *The Accounting Review* 77(2): 379-395.
- [29] Pfeiffer, T., U. Schiller, and J. Wagner. 2011. Cost-based Transfer Pricing. *Review of Accounting Studies* 16(2): 219-246.
- [30] Prendergast, C. 2002. The Tenuous Trade-Off between Risk and Incentives. *The Journal of Political Economy* 110(5): 1071-1102.
- [31] Rogerson, W. 1992. Contractual Solutions to the Hold-up Problem. *Review of Economic Studies* 59: 777-793.

- [32] Ross, S. 2004. Compensation, Incentives, and the Duality of Risk Aversion. *Journal of Finance* 59: 207-225.
- [33] Schmidt, K. 2003. *On the Covariance of Monotone Functions of a Random Variable*. Lehrstuhl für Versicherungsmathematik, working article, Technische Universität Dresden.
- [34] Williamson, O. 1985, The Economic Institutions of Capitalism. *New Press*.
- [35] Wilson, R. 1968. On the Theory of Syndicates. *Econometrica* 36: 119-132.
- [36] Ziv, A. 2000. Information Technology and Optimal Firm Structure. *Journal of Accounting Research* 38(2): 297-328.